

THE STABILITY OF STONE ON A SUBMERGED SPUR CREST

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by

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## SYNOPSIS

A spur is a low height earth or rockfill projection extending from the bank of a stream into the channel. It may be used to redirect low flow currents, to stabilize the spur-side bank, to maintain a deeper channel opposite the spur, or to create a backwater upstream. A typical spur or groin is usually overtopped during high flood stages. During the time the spur is submerged, erosive forces will be acting on the crest. The purpose of this work is to define the rock size required on the crest for stability.

This problem is complex and required physical modelling. A simple relationship for the required stone size was developed as a function only of the difference in water depths upstream and downstream of the structure. The difference in water levels, or backwater caused by the spur, was found to be a function of the contraction ratio, the downstream Froude number, and the ratio of the upstream flow depth to spur height.

Given a stage-discharge relationship for the stream and the geometry of the river and spur, the backwater effects may be predicted and the required stone size determined.

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## CHAPTER 1

### INTRODUCTION

A spur or groin is an earth or rock fill embankment which extends into a stream channel. It is built to control the direction of water and/or sediment movement, for the purpose of river training, bank protection, land reclamation, or channel improvement. It is usually a low height structure, both for economy and to minimize backwater effects during flood stage. Because of its low height, the spur may frequently be overtopped during times of flood stage in the stream.

When the spur is submerged, erosive forces will be acting on it. These forces are usually greatest at the nose of the spur, but are also significant on its crest. To keep the structure intact, these forces must be resisted. One common method of resisting such erosion is to place rock on the embankment.

For the problem of erosion of the crest of the spur, which is the subject of this thesis, very little published work has been found. The purpose of this work was to develop a procedure for the selection of an appropriate rock size for the crest of a given spur.

The amount of rock required for the crest protection of a spur is not large, so the economics of the project will not allow much time for its engineering design. Therefore, an important consideration in this work was that the final result must be a simple and quick design procedure, even at the expense of some precision.

A theoretical analysis of this problem is very difficult because of the complex interaction of forces acting on the stone in a highly curved flowfield, and the unknown magnitude of the component of the discharge which passes over the spur. Because of this complexity, physical model testing was required.

Three sets of experiments were done. In the first set, the objective was to find a functional relationship for the stability of the stone on a two-dimensional spur crest in terms of measurable variables. In the second set, the hydraulics of the flow over and around a submerged spur were studied. A third set of tests was done to check that the results of the first two sets could be combined in a three-dimensional situation.

The tests were done in three rectangular flumes at the University of Saskatchewan. They were limited to the case of a straight spur projecting perpendicularly



into a flow where the velocity distribution upstream is uniform across the width of the river. Spurs constructed entirely of a uniform rock fill were used for all of the tests, but the results should have application to graded rock fills and to earth fill embankments which have a layer of rock on the crest.

## CHAPTER 2

### EXPERIMENTAL APPARATUS

#### A. The Tilting Flumes

Two tilting flumes at the University of Saskatchewan were used for most of the tests (Fig. 1,2). The flumes are very similar; each has a brass floor, glass walls, and an adjustable overflow tailgate. The smaller flume is 305 mm (1 foot) wide and 9 m long. The other is 800 mm wide and 10 m long. The discharge in each flume is measured with an orifice meter in the supply line. The upstream and downstream water levels were measured with a movable point gauge. The slope of each flume was set at zero for all tests.

#### B. The 2.5 m Flume

To obtain a more realistic channel width to flow depth ratio for the second and third sets of tests, a wider flume was needed. A 2.5 m wide flume 18 m long was built of concrete blocks on the lab floor (Fig. 3). A sheet of 6 mil poly was laid inside the blocks for watertightness. A head tank was built in the same way (Fig. 4). Frames on which point gauges were mounted were installed 1.5 m upstream of the model and 9.2 m downstream. The two gauges were correlated using still water readings. The tailwater level was controlled by



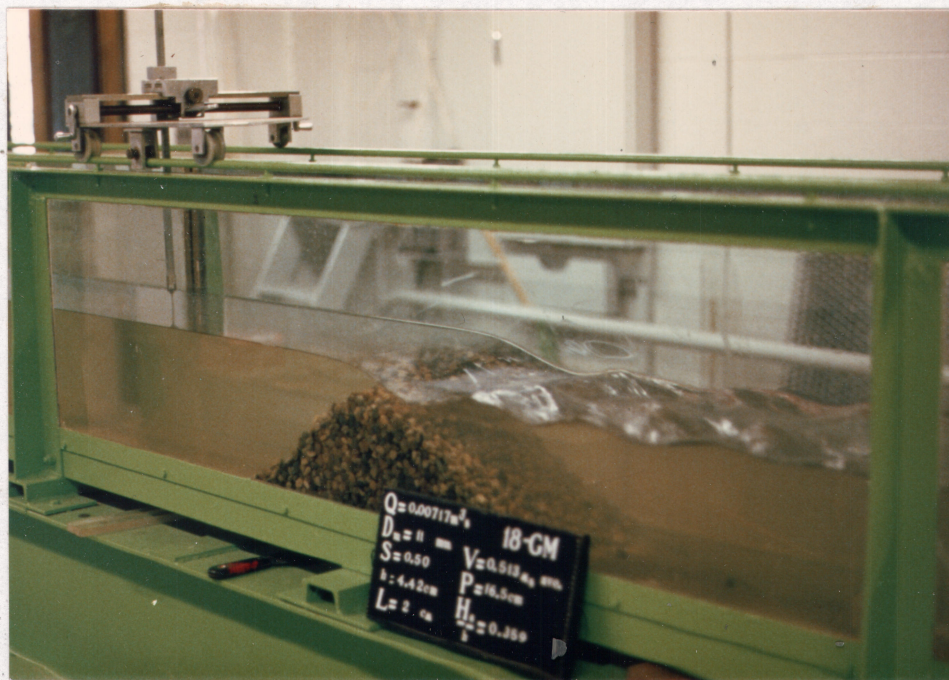


Fig. 1. Typical rock model in the small (305 mm) flume.



Fig. 2. Downstream view of a typical full-width rock model in the 800 mm flume.



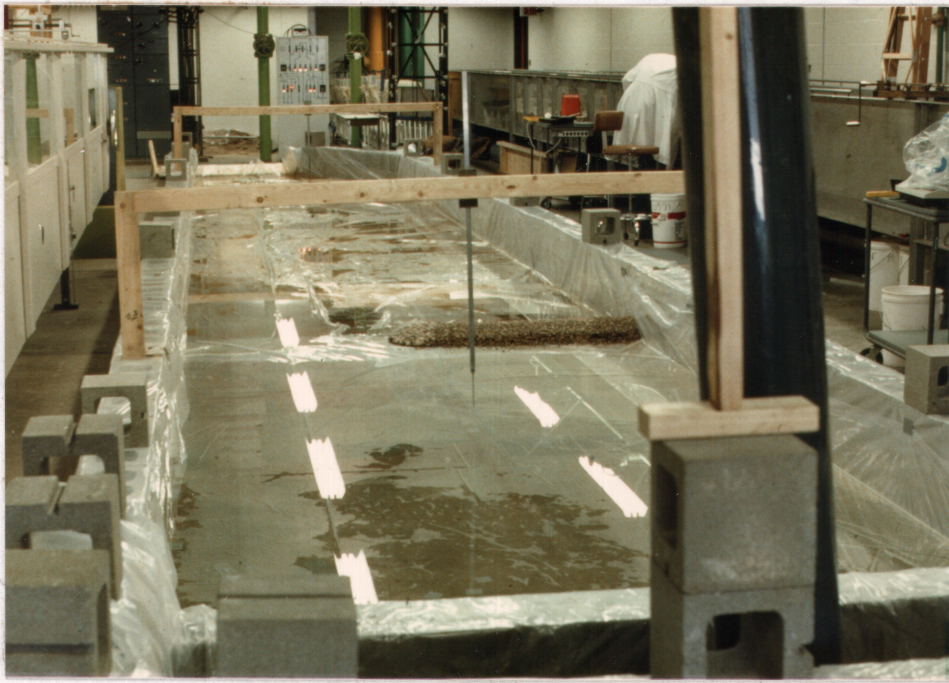
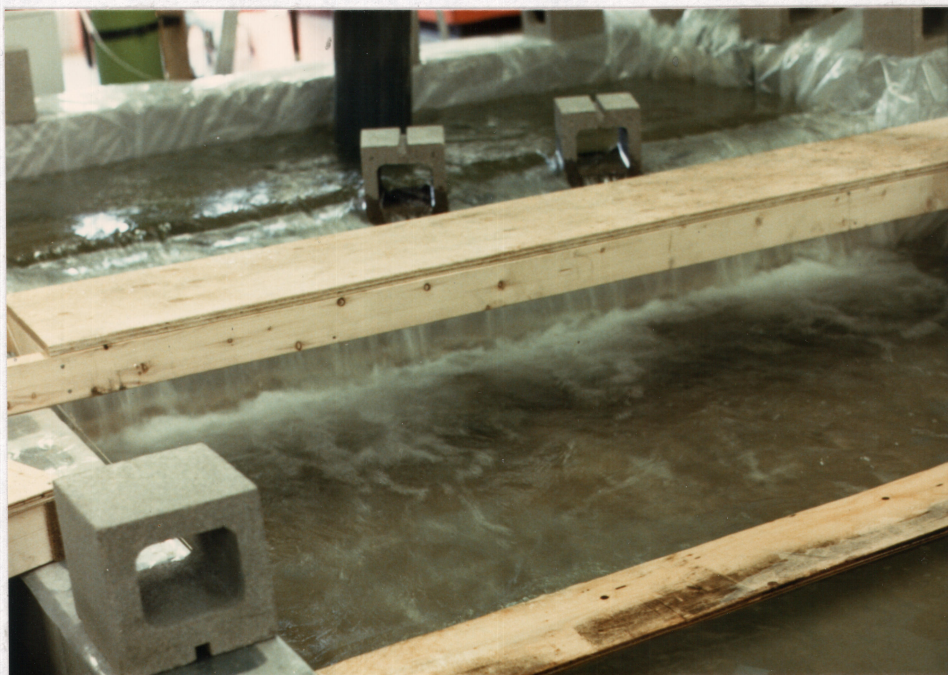


Fig. 3. The 2.5 m flume showing point gauge frames and a typical model spur.



weir  
walkway

wave  
inhibitor

Fig. 4. The head tank for the 2.5 m flume. The blocks on the front wall of the tank were needed to create a uniform lateral velocity distribution; the floating plywood was used to inhibit wave action.



an overflow tailgate made of a 50 mm by 200 mm board on edge.

The downstream point gauge was mounted as far downstream as possible to include both the head loss caused by the spur and the recovery due to the deceleration in the test reach. This was not accomplished completely; in some cases the eddy downstream of the spur extended beyond the location of the point gauge. In fact, Francis et al. (1968) found that the eddy downstream of a spur on an alluvial bed extended about 13 spur lengths downstream for contraction ratios between 0.10 and 0.25, and oscillatory motion was observed as far as 40 lengths downstream. Since the model spur lengths in these tests ranged from 0.6 m to 1.25 m, a 9 m reach downstream of the spur would not be enough to exactly obtain uniform flow in every case. However, most of the deceleration of the jet occurs in the gauged reach, and since both the head loss and the recovery are functions of the square of the jet velocity, the error in the readings due to the reach being too short is very small.

The concrete lab floor which formed the bed of the flume was not level longitudinally or transversely. There was, however, a fairly level reach in the center third of the profile, so the model was placed at the upstream end of this level section to reduce the effect of the irregularity. The brick wall forming the front

overflow section of the head tank was not level, and this led to problems in obtaining a uniform velocity distribution across the channel. A uniform velocity distribution was desired to eliminate this effect as a variable in the test program. It was obtained by trial by placing concrete blocks at intervals on the front wall of the head tank until the flow, as judged by observing dye patterns, was uniformly distributed across the channel.

Several tests done without a model in the flume showed that the boundary friction was negligible at typical values for discharge and depth. There there was no measurable head loss between the point gauges in any of these tests.

### C. The Models

The rock used for the models was subrounded to subangular stone of uniform gradation. The angle of repose and specific gravity of the rock are given in Table 1. The angle of repose was found by dumping a pile of stone onto a level surface and measuring the slope of the sides of the cone in several places. Specific gravity was determined by submerging a weighed sample of stone to find its volume.

TABLE 1

Properties of Stone Used for Tests

Seive Passing (mm)	Size Retaining (mm)	Nominal $d_m$ (mm)	Angle of Repose (degrees)	Specific Gravity
4.75	9.52	7.14	36	2.67
9.52	12.7	11.1	39	2.72
12.7	19.0	15.9	38	2.68
19.0	25.4	22.2	42	2.71

The models were formed by dumping the rock into place and lightly tamping the crest and downstream slope to the required elevation and slope. This was to simulate construction of a real spur constructed by end dumping, resulting in some compaction of the crest. The upstream slope was left at the angle of repose.

The crest elevation of the rock embankment was measured by slowly filling the flume with water and recording the water level when the water was about 0.2 to 0.3  $d_m$  below the top of the top stones. Fig. 5 illustrates the procedure. In case (a) the water level is just below the crest; (b) shows the point gauge measurement of the water surface at the crest elevation; and (c) shows the crest slightly submerged. This procedure was somewhat subjective but was fairly repeatable. There was only a small difference in water

surface elevation between cases (a) and (c).

Larger stone was hand placed on the end of the spur in some of the three-dimensional tests to prevent erosion of the nose.





(a)



(b)

Fig. 5. Determination of crest elevation. (a) water level slightly below the crest; (b) measurement of water surface elevation at crest level with point gauge. (Continued on following page.)





(c)

Fig. 5. Determination of crest elevation. (c) water level above the nominal crest.

## CHAPTER 3

### STABILITY TESTS

#### A. Background

Several different methods and examples exist for choosing the design size of rock riprap for bank protection (U.S. Bureau of Public Roads, 1967; California Division of Highways, 1970; Stephenson, 1979) or the design size of rock for weirs or dams built underwater (Straub, 1953; Izbash and Khaldre, 1959). In general, these methods are based on the velocity of water either in the stream or directly against the stone. The relevant velocity may sometimes be easily determined, but for the case of flow over a spur, it is difficult to predict the velocity of the water involved because of the unknown magnitude of the discharge over the spur. Two possible approaches to this problem were considered: (1) Resolve the discharge into its components over and around the spur; or (2) develop a relationship for the stability of the stone which does not explicitly include the unit discharge over the spur. An attempt was made to divide the discharge but this was found to be too difficult because of the number of variables involved. Therefore, the objective of the first set of tests was to find the stable stone size in terms of some other variables which could be readily measured or determined.



## B. Theory

The limiting minimum downstream submergence  $h_s$  for stability of the stone on the crest was expected to be a function of the hydraulic conditions, the geometry of the spur, the fluid properties, and the stone properties. The hydraulic conditions are defined by the upstream head  $h$  and the acceleration of gravity  $g$ . Spur geometry includes the spur height  $P$ , the crest length  $L$  parallel to the flow, and the downstream slope of the spur embankment  $S$ . Fluid properties include the fluid density  $\rho$  and the kinematic viscosity  $\nu$ . Stone properties include the median rock diameter  $d_m$ , the stone density  $\rho_s$ , and the gradation of the stone sample. Some of these variables are shown in the definition sketch in Fig. 6. For these tests, the gradation of the stone was eliminated as a variable by using a uniform gradation of stone in each case. Symbolically, then

$$\text{minimum } h_s = f(h, g, P, L, S, \rho, \nu, d_m, \rho_s) \quad \dots(1)$$

By deductive reasoning the minimum  $h_s$  would be expected to increase for a larger  $h$ , since a larger  $h$  indicates a higher velocity. It would also be expected to increase for a smaller  $d_m$ , since a smaller rock has less resistance to movement. The effects of  $P$ ,  $L$ , and  $S$  are less obvious, but as components of the geometry they were included in the test program to study the effect, if any, of a change in these variables.

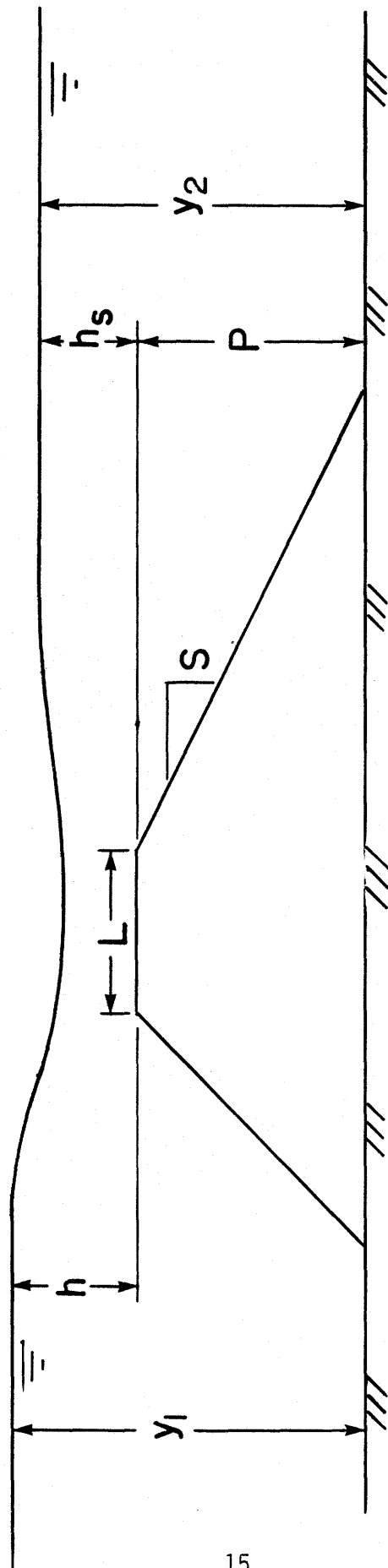


Fig. 6. Definition sketch - Spur profile.

The effect of a change in the fluid density  $\rho$  was not included in these tests, since the density of fresh water is practically constant. If a denser fluid was used, the minimum  $h_s$  would be increased because of the increased lift, drag, and shear forces associated with the denser fluid.

The rock density was evaluated by considering the specific gravity of the rock  $S_g$ , or the ratio of densities of rock and water. The specific gravity of the stones in the tests was near 2.7, as shown in Table 1. This is a very common value that occurs frequently in practice. The effect of a different specific gravity was not considered experimentally, but the results of this study may be used for stone with a specific gravity different than 2.7 by multiplying the design stone diameter by the factor  $(2.7-1)/(S_g-1)$ .

The Reynolds number for each of the tests was greater than  $10^4$ , for which the effect of viscosity can be omitted. Since all applications of the study are earth based, the acceleration of gravity was not considered to be a variable, nor was it required for the formation of non-dimensional groups.

Making the remaining terms non-dimensional, using  $h$  as the characteristic length, gives

$$\text{minimum } \frac{h_s}{h} = f\left(\frac{d_m}{h}, \frac{P}{h}, \frac{L}{h}, S\right) \quad \dots(2)$$

The effect of the last three terms in equation (2) is expected to be small compared to the first term. The spur height P would only influence the stability by its effect on the approach velocity, and so the velocity of flow on the crest. This is a very small effect. The crest length L influences the amount of the upstream head which is dissipated by friction, and this again is a small effect for practical lengths. It is difficult to see that the downstream slope would have any effect at all on the stability of rock on the crest. These terms are neglected for the remainder of this analysis.

It may be shown that for fully turbulent flow with a given fluid density and a given rock density, the rock diameter at incipient instability is a function of the velocity squared. The forces producing instability, that is lift, drag, and shear, are each functions of the velocity squared and the area on which they act, which is proportional to the square of the rock diameter. The stabilizing force, that is the submerged weight of the rock, is a function of the volume of the stone, or the diameter cubed. At incipient instability, these forces are equal; hence

$$v^2 d_m^2 \propto d_m^3 \quad \dots(3)$$

from which

$$d_m \propto v^2 \quad \dots(4)$$

This relationship is well established in practice. For example, for riprap on a channel bed downstream of a stilling basin the United States Bureau of Reclamation (USBR) recommends (Peterka, 1964, p. 208)

$$d_m = 0.041 v^2 \quad \dots(5)$$

where  $v$  is the bottom velocity in the channel in m/s and  $d_m$  is the diameter of a particle in m. This assumes particles with a specific gravity of 2.65.

Similarly, for riprap at a culvert outlet, Smith (1978) recommends

$$d_m = 0.019 v^2 \quad \dots(6)$$

as a conservative value. Here  $v$  is the average outlet velocity in m/s.

In the case of flow over a submerged spur, an indication of the magnitude of the velocity produced is given by the drop in water level across the spur. The average velocity, assuming the entire drop is converted to velocity head, may be taken as

$$v = \sqrt{2 g \Delta h} \quad \dots(7)$$

where  $h$  is the drop in water surface elevation. The actual velocity over the spur will be somewhat greater than this because the velocity head of the approach flow is neglected. Ignoring any recovery of velocity head downstream from the spur, this drop in elevation would be

$$\Delta h = h - h_s \quad \dots(8)$$

from which



$$v^2 \propto h - h_s \quad \dots(9)$$

Combining (4) and (9) yields

$$d_m \propto h - h_s \quad \dots(10)$$

This equation suggests that there will be a linear relationship between required rock size and difference in water levels across the submerged spur.

An indication of the possible magnitude of the constant of proportionality in equation (10) may be determined using the coefficient in Smith's equation (6). Substitution of equation (8) into equation (7), and equation (7) into equation (6), yields

$$d_m = \frac{h - h_s}{2.7} \quad \dots(11)$$

### C. Procedure

Tests were done over a range of values for each of the variables in equation (2) to determine the functional relationship. Most of the tests were done with small values of  $L/h$  (Fig. 7) and  $P/h$  (Fig. 8) but several tests were done with larger values of each of these variables. The downstream slope of the embankment was 0.5 for most tests but some were done with  $S$  at 0.33 or 0.2. Four stone sizes were used, as was shown in Table 1.

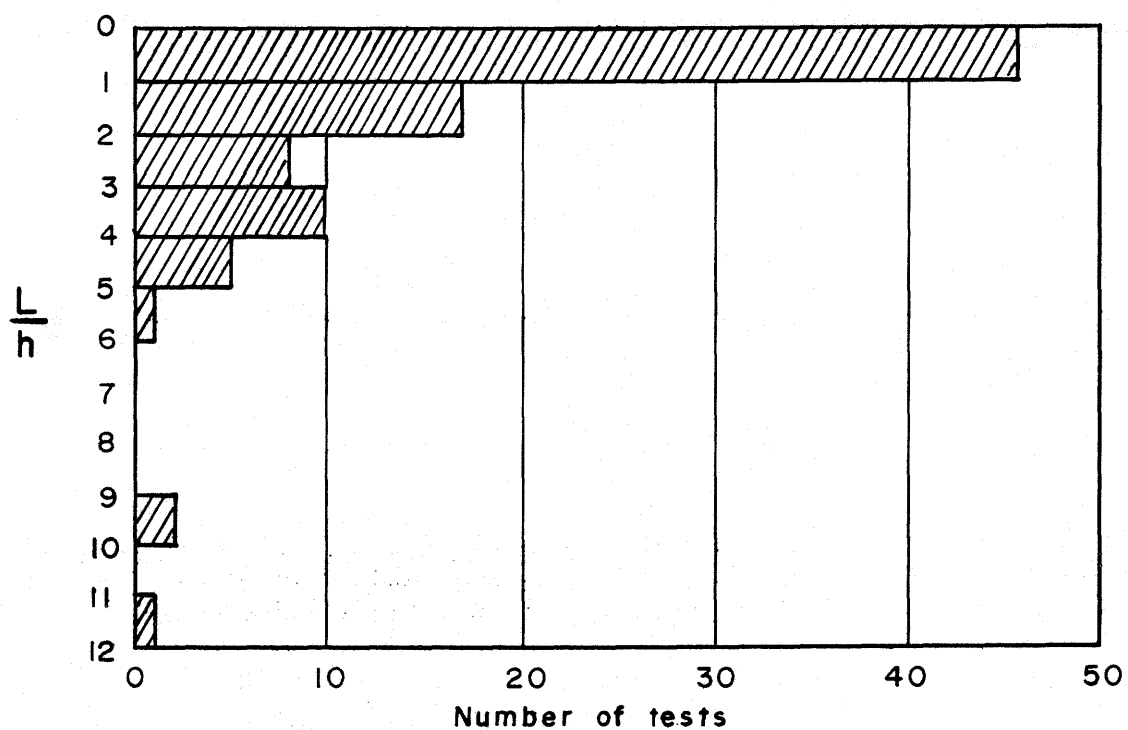


Fig. 7. Range of values of  $L/h$ .

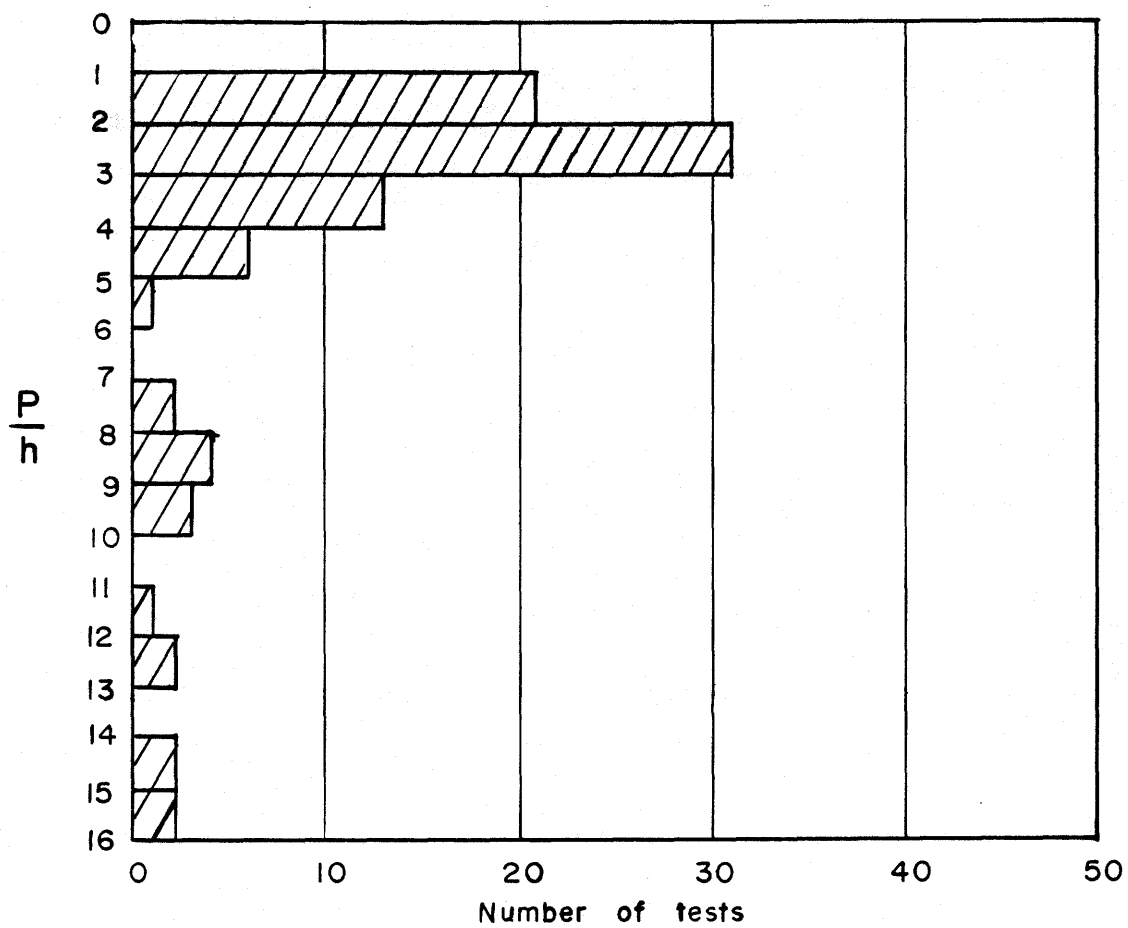


Fig. 8. Range of values of  $P/h$ .

The procedure for the tests was as follows:

1. Build a rock spur across the entire width of the small flume.
2. Record the crest length of the spur (parallel to the flow), the downstream slope of the spur, and the rock size.
3. Fill the flume with water, recording the crest elevation as described in Chapter 2.
4. Set the required discharge in the flume.
5. Lower the tailgate by small increments, watching for rock movement at the crest.
6. At instability, record the upstream and downstream water elevations.

This procedure was repeated, varying the rock size, discharge, crest length (parallel to the flow), downstream slope and height of the embankment (Fig. 9 - 12). In all, 90 tests were run.

Several different criteria for instability were used. "First movement" was defined as the complete dislodgement of a representative stone. There were a few thin, flat stones in the sample; these tended to move very early and unpredictably depending on their orientation to the flow. The movement of these stones was not





Fig. 9. Typical rock spur at first movement.  $q = 0.0247 \text{ m}^2/\text{s}$ ,  $d_m = 15.9 \text{ mm}$ ,  $P = 127 \text{ mm}$ ,  $S = 0.50$ ,  $L = 30 \text{ mm}$ .



Fig. 10. Rock spur with small discharge in 800 mm flume at first movement.  $q = 0.0132 \text{ m}^2/\text{s}$ ,  $d_m = 15.9 \text{ mm}$ ,  $P = 254 \text{ mm}$ ,  $S = 0.50$ ,  $L = 50 \text{ mm}$ . In this case  $h_s/h$  is negative because the tailwater level is below the crest.



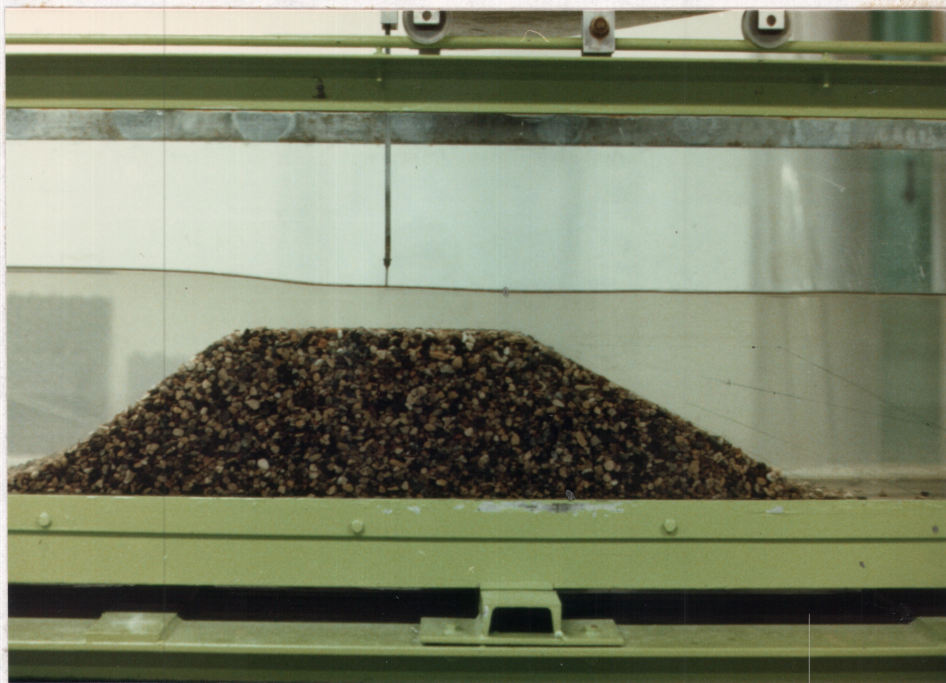


Fig. 11. Rock spur with longer crest, at first movement. Movement occurred at the downstream edge of the crest.  $q = 0.0247 \text{ m}^2/\text{s}$ ,  $d_m = 7.1 \text{ mm}$ ,  $P = 174 \text{ mm}$ ,  $S = 0.50$ ,  $L = 300 \text{ mm}$ . (Run 41-FM)



Fig. 12. Rock spur with flatter downstream slope.  $q = 0.0251 \text{ m}^2/\text{s}$ ,  $d_m = 7.1 \text{ mm}$ ,  $P = 176 \text{ mm}$ ,  $S = 0.20$ ,  $L = 20 \text{ mm}$ .



considered to be "first movement". The stones tended to rock back and forth or roll slightly in their niches, but first movement was not considered to have occurred until one stone was completely removed from its original position.

In an attempt to find a criterion which involved less random variation, "second movement" was defined as the second dislodgement of a representative stone. It was anticipated that this would be more repeatable than first movement.

"Group movement" was defined as the first movement of a group of stones all at once. This would be closer to what would be considered "failure" in the prototype.

All three of the criteria mentioned above were used in recording the data for most of the tests (Fig. 13). For analysis, one had to be selected. This selection did not need to be based on which condition was closest to a field definition of failure, since this could be taken care of by the selection of a factor of safety. The most important consideration was the consistency or repeatability of the results. A simple analysis showed that, on this basis, "first movement" was the best choice. First movement was used as the condition for instability for the analysis of the results of the first set of tests, and was the only condition recorded for the second and third sets.





(a)



(b)

Fig. 13. Progress of a typical test; constant discharge, gradually decreasing tailwater depth.  $q = 0.0249 \text{ m}^2/\text{s}$ ,  $d_m = 7.1 \text{ mm}$ ,  $P = 168 \text{ mm}$ ,  $S = 0.50$ ,  $L = 70 \text{ mm}$ . (a) First movement; (b) Second movement. (Continued on the following page.)





(c)

Fig. 13. Progress of a typical test. (c) Group movement. Note visible rounding of downstream edge of crest.



In general, movement occurred first at the downstream edge of the crest. The dislodged stone would tumble a short distance down the slope before coming to rest. If the test was continued beyond the first signs of movement, the eroded area would expand upstream into the crest and down the downstream slope.

#### D. Results

The detailed results of the 90 tests in the first set are given in Appendix B. A statistical analysis of the results showed the expected correlation between  $h_s/h$  and  $d_m/h$ . However, over the range of values tested, the other variables (weir height  $P$ , crest length  $L$ , and downstream slope  $S$ ) did not have a significant effect on the stability of the rock on the crest.

A multiple regression analysis using the SPSS<sup>™</sup> program on the DEC 2060 computer at the University of Saskatchewan yielded the equation

$$\frac{h_s}{h} = 1.085 - 1.51 \frac{d_m}{h} - 0.013 \frac{L}{h} - 0.12 S + 0.0029 \frac{P}{h} \quad \dots(12)$$

with a correlation coefficient "r" of 0.950. The small magnitudes of the last three terms, considering typical values taken by the variables, indicate their relative unimportance. Ignoring them yields the equation

$$\frac{h_s}{h} = 1.02 - 1.50 \frac{d_m}{h} \quad \dots(13)$$

with only a marginally poorer correlation coefficient

of 0.947. This confirms the expected dominant effect of  $h$  and  $d_m$ , and shows that  $P$ ,  $L$ , and  $S$  have no real effect on the stability of the stone on the crest.

For design purposes, equation (13) may be approximated by

$$\frac{h_s}{h} = 1.0 - 1.5 \frac{d_m}{h} \quad \dots(14)$$

which can also be written as

$$d_m = \frac{h - h_s}{1.5} \quad \dots(15)$$

This suggests that incipient instability (defined as first movement) of the rock at the crest occurs when the difference between upstream and downstream water levels exceeds  $1.5 d_m$ .

A similar analysis was done using the other criteria for instability. For second movement, the denominator of the right hand side of equation (15) would be 1.8; for group movement it would be 2.8.

Equation (15) is reasonable in that if the water is still,  $h_s = h$  and the stone size required for stability is 0. As the differential head increases, the required stone size increases. The results of the tests are seen in Fig. 14.

In the tests, the discharge was held constant while the tailwater was gradually lowered until movement occurred. Initially, the upstream and downstream water levels are close to the same and the velocity on the crest is low. As the tailwater level drops, the

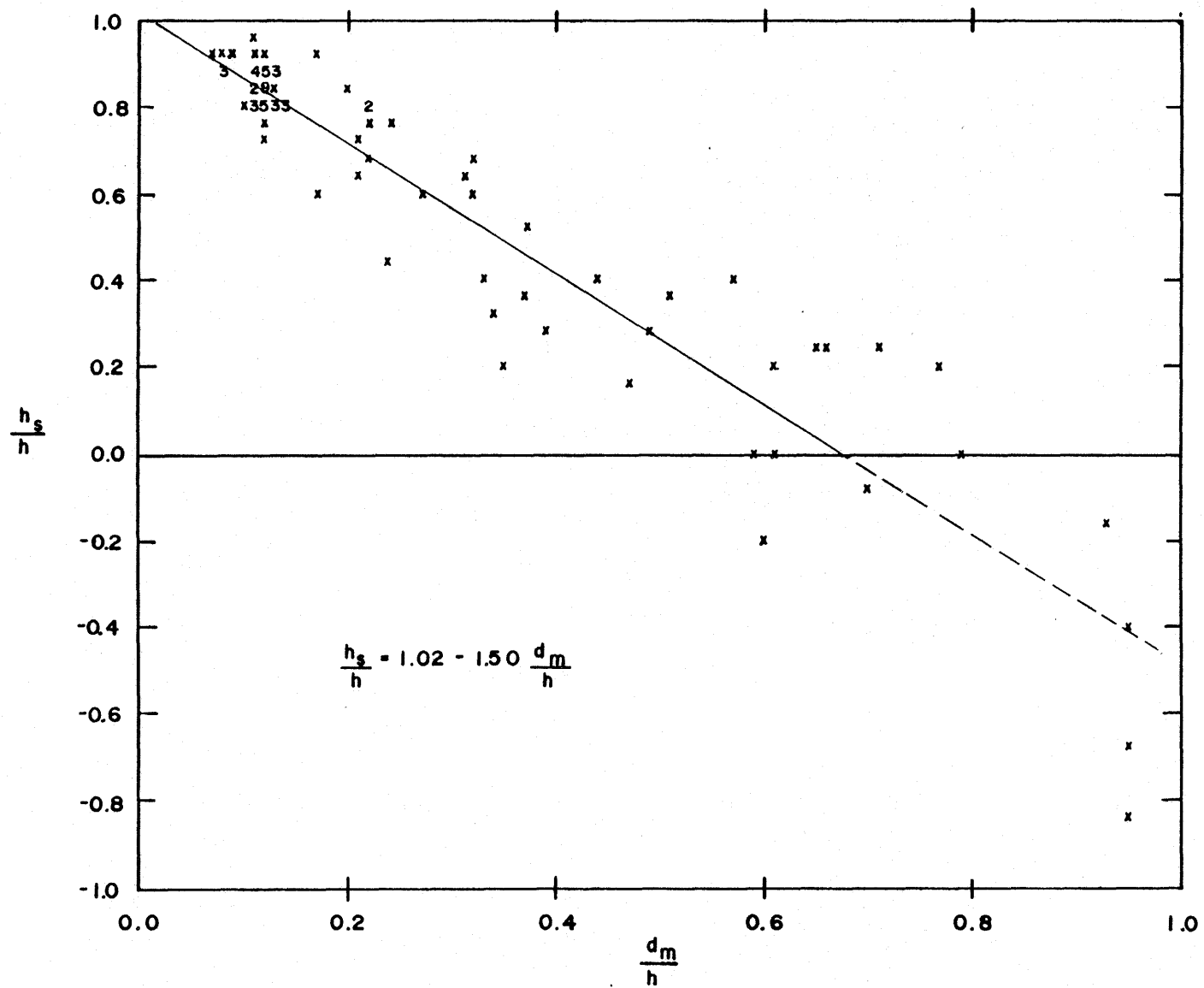


Fig. 14. Rock size required for stability.

headwater falls and the depth of water on the crest also decreases, while the velocity there increases. Fig. 11 and Fig. 13(a) show typical profiles of this condition.

Eventually critical depth occurs on the crest. Beyond this point, further lowering of the tailwater cannot change the conditions upstream of the section of critical depth, except for a slight lowering of headwater due to increased throughflow. However, the depth at the downstream end of the crest, and so the velocity at that point, are still dependent on  $h_s$ . This is illustrated in Fig. 13(b) and (c). The highest velocity must occur where the depth of flow is least; that is, at the downstream end of the crest. The forces of lift, drag, and shear, which are all functions of the square of the velocity, are also highest at this point. Depending on the discharge and rock size, instability may occur at any of these stages.

If the discharge is small and the rock size is relatively large, instability may not occur until after the tailwater level has fallen below the crest. This is the case for the points below the line  $h_s/h = 0.0$  in Fig. 14, and is illustrated in Fig. 10. Extension of the regression line through these points is problematical, and for this reason the line is dashed in this region. When  $h_s$  is negative, it theoretically should not influence the depth of water on the crest, the

velocity at the downstream edge, or the stability of the rock on the crest. If the tailwater was low enough that flow would occur down the downstream slope, then its level would again become important. This was not the case in any of these tests. The points in the right half of Fig. 14 represent very small values of  $h$ . For example, for run 88-FM, where  $d_m/h = 0.948$  and  $h_s/h = -0.855$ ,  $h$  is only 16.8 mm. However, it is possible that the downstream depth does influence the rock stability in such a case because the water level is less than one rock diameter below the crest.

The region of Fig. 14 where  $h_s/h$  is less than zero contains a large amount of scatter and equation (15) may not apply. However, this region is not of much practical importance because of the different sequence of events in the prototype. Here the discharge is not constant, but increases in some unique relationship with the tailwater depth. Initially, as the discharge increases to the point at which the spur is first overtopped, the downstream water level will be below the crest and  $h_s$  will be negative. As the discharge increases, both the headwater and tailwater depths increase. The upstream depth depends on the downstream depth and the geometry of the situation, as discussed in Chapter 4. The downstream depth is given by the stage-discharge relationship. Thus, each situation is unique, but it is not likely that the largest differ-

ence between upstream and downstream depths will occur while the spur is overtopped and  $h$  is negative. The critical case will usually be at some larger discharge, for which equation (15) applies without question.

A comparison may be made between the different values of the denominator of the right hand side of equations (11) and (15). When one of these equations is used for design, a smaller value of the denominator is more conservative because it yields a larger rock size for the same conditions.

Smith's value of 2.7 is much less conservative than the value of 1.5 noted for first movement in the present work. This is reasonable because of the relatively minor consequences of failure of the riprap below a culvert outlet; because first movement at a culvert outlet would not be considered failure; and because a rock would be more difficult to remove from a basin or even a level bed than from the crest of a spur for the same incident velocity. On the other hand, the value of 2.7 compares well with the value of 2.8 noted for group movement in this study.

The values found in this work, then, compare well with Smith's recommendation. For design, the criterion of first movement and the value of 1.5 may be too conservative. Since "group movement" was defined as the first occurrence of simultaneous movement of three or

more stones, even the value of 2.8 may be too conservative in some cases. This issue is discussed in chapters 5 and 6.

## CHAPTER 4

### HYDRAULIC PERFORMANCE TESTS

#### A. Rationale

In the first set of tests a relationship was developed between the required rock size and the difference in water levels across the structure. To make this relationship useful, it was necessary to find the difference in water levels as a function of variables which would be known at the design stage in a real situation. These variables are the discharge  $Q$  corresponding to various downstream depths  $y_2$ , the river width  $B$ , the size of the opening  $b$ , and the height of the spur  $P$ . A plan view definition sketch is shown in Fig. 15. The shape of the spur is another variable but for this project only a straight spur projecting perpendicularly into the flow was considered.

#### B. Background and Theory

Kindsvater and Carter (1955) showed that for a rectangular, inerodible channel,

$$C_D = f\left(\frac{b}{B}, F, \frac{L}{b}, \frac{r}{b}\right) \quad \dots(16)$$

where  $F$  is the Froude number in the contracted section,  $r$  is the radius of the corners of the embankment, and  $C_D$  is the coefficient of discharge in the equation

$$Q = C_D b d_2 \sqrt{2g(\Delta h - h_f + h_{v1})} \quad \dots(17)$$

In equation (17)  $d_2$  is the depth of the contracted jet,  $g$  is the acceleration of gravity,  $b$  is the width of the



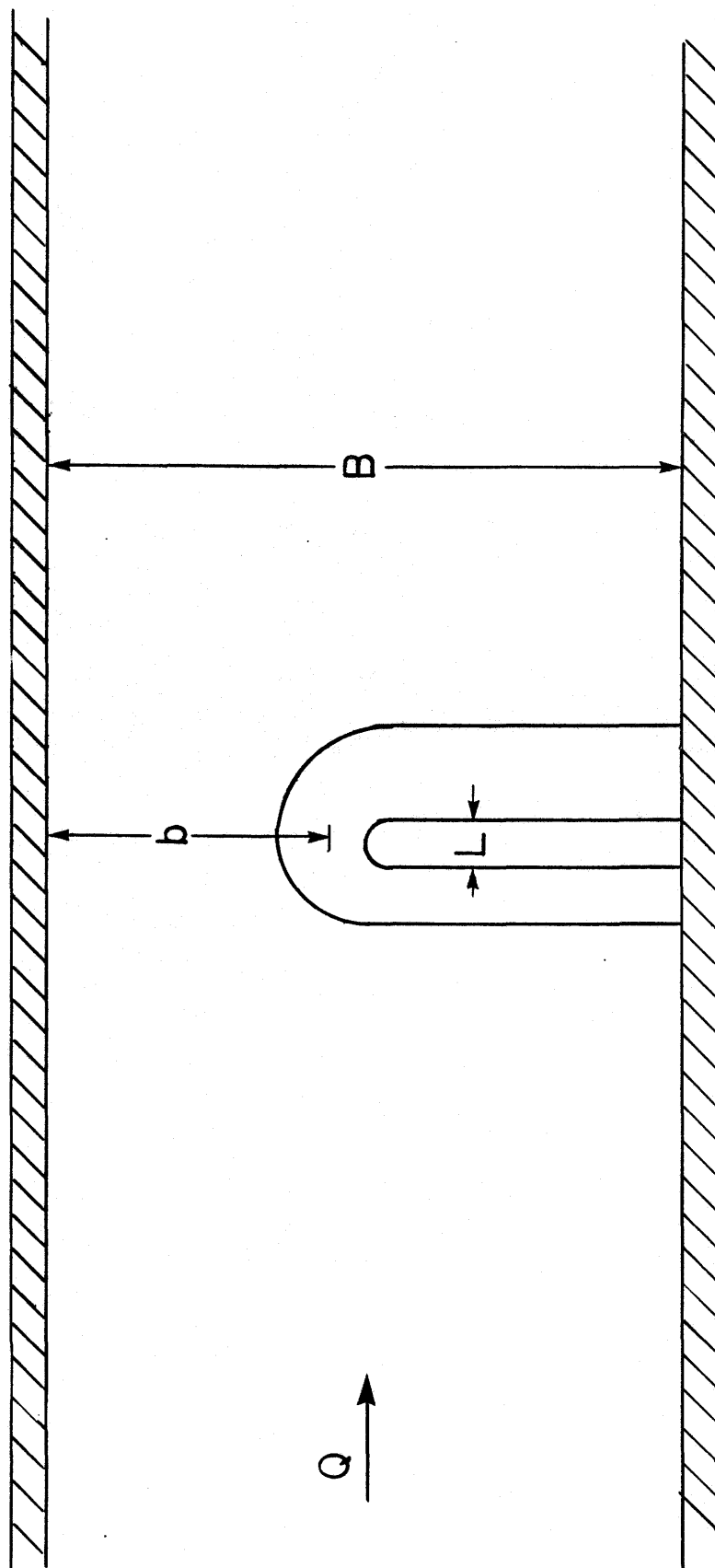


Fig. 15. Definition sketch - plan view of spur.

opening as before,  $\Delta h$  is as shown in Fig. 16,  $h_f$  is the friction loss in the reach, and  $h_{v1}$  is the upstream velocity head.

Tracy and Carter (1955) defined the "backwater ratio"  $\frac{h_1^*}{\Delta h}$  and showed that it is a function of the contraction ratio  $m$  ( $m = 1 - b/B$ ), the Froude number in the contraction  $F$ , the coefficient of contraction  $C_c$ , and the roughness, expressed by Manning's 'n'.

$$\frac{h_1^*}{\Delta h} = f(m, F, C_c, n) \quad \dots(18)$$

This work has been used as the basis of the contracted area method of determining discharge (Chow, 1959) and it is well suited for this situation. If the water surface profile is known it is relatively easy to compute the discharge. However, if the discharge is known and the backwater is to be determined, this method is not as useful. As Henry (1955) pointed out, the actual elevation of the upstream water surface elevation must involve a trial and error solution for  $\Delta h$ . In addition, an assumption must be made for the coefficient of contraction to get the contracted width of the jet.

Izzard (1955), in his discussion of Tracy and Carter's work, suggested the use of the depth ratio  $y_1/y_n$  instead of the backwater ratio  $\frac{h_1^*}{\Delta h}$  where  $y_1$  is the upstream depth and  $y_n$  the downstream, (assumed to be) normal depth. Using their data, he found that  $y_1/y_n$

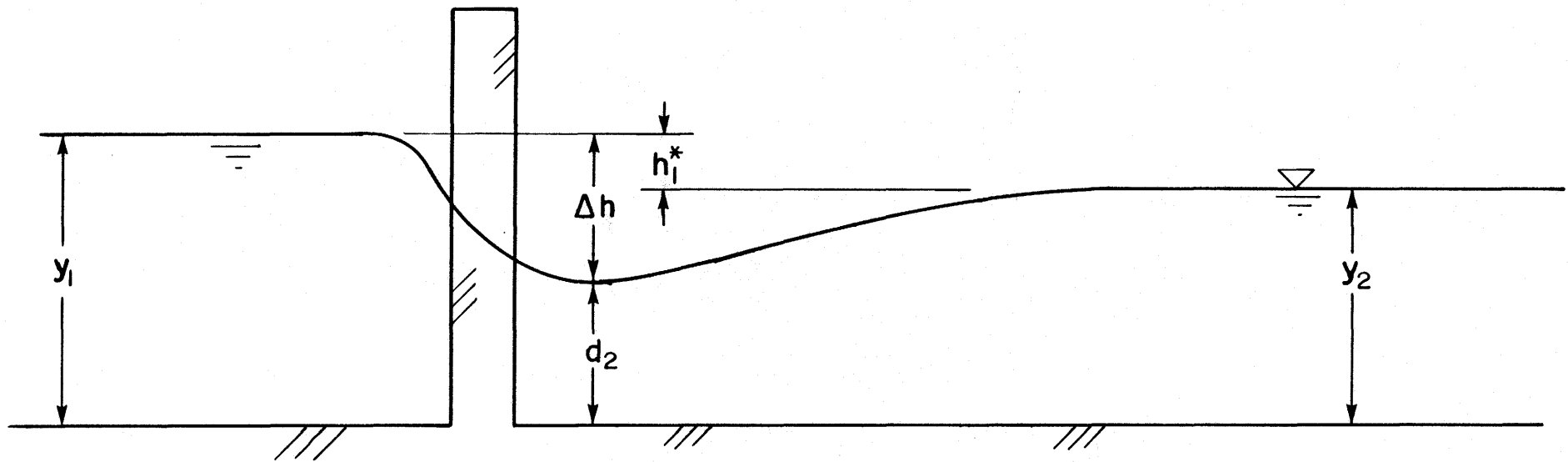


Fig. 16. Backwater at a constriction assuming no friction. After Tracy and Carter, 1955.

was a non-linear function of  $\frac{v^2}{2gy_n}$ . Izzard's suggestion was acted on by Sandover (1970), in his work on backwater effects of a cofferdam constructed by end-dumping. He found that

$$\frac{h_1^*}{y_n} = f(F_2, m) \quad \dots(19)$$

where  $F_2$  is the downstream Froude number and  $y_n$  is the normal depth in the stream, which is assumed to be equivalent to the depth downstream of the spur. This work was done on an erodible bed with  $F_2$  in the range 0.08 to 0.125. The use of the downstream Froude number rather than the Froude number in the contracted section greatly simplifies the calculations, since the downstream conditions are known before the spur is built.

Das and Nuttal (1973) confirmed the existence of this relationship for Froude numbers of 0.10, 0.29, and 0.50. They also pointed out that backwater rise on an alluvial bed is much less than for a rigid bed because of scour effects.

The work of all of these investigators involved constricting elements extending from both banks, with an opening in the center of the stream. A spur extending from only one bank, as in the present work, could be seen as a half model of the same situation. This would be valid provided that the bank comprising the centerline of the half model was frictionless. In the model tests performed here, this condition was essentially satisfied.

### C. Procedure

For these tests, a rock spur was constructed to a specified height and length. A large stone size was used to ensure that the spur would not erode. The flume was then filled with water, and a certain discharge was set at the valve. The tailgate was lowered incrementally, and upstream and downstream point gauge readings were recorded in each case after steady state was attained.

Twenty-four tests of this type were done in the 800 mm flume. To test a different width to depth ratio, the 2.5 m flume was built and nineteen tests were performed in it. The tests included a variety of spur heights, contraction ratios, and discharges. All of the embankments were built with downstream slopes of 0.5.

### D. Results

The detailed results of the tests are contained in Appendix C.

The analysis involved finding the form of the relationship

$$\frac{y_1}{y_2} = f\left(F_2, \frac{b}{B}, \frac{y_1}{P}\right) \quad \dots(20)$$

The first two independent variables are the same as in equation (19). The third term,  $y_1/P$ , was added to account for the discharge over the spur, which was not present in earlier work.



In general, the backwater depth ratio  $y_1/y_2$  increases with increasing downstream Froude number as shown in Fig. 17 for the case of negligible flow over the top of the spur ( $y_1/P < 1.05$ ). The backwater depth ratio must be 1.0 at a Froude number of 0.0 corresponding to still water.

Fig. 17 also shows the effect of contraction ratio on the backwater depth ratio. As would be expected, the backwater depth is less for a larger opening at the same downstream Froude number.

Fig. 18, 19 and 20 show the effect of different spur heights. As the  $y_1/P$  ratio increases, the discharge over the spur increases. For the same downstream Froude number there is less discharge through the opening, less head loss in the contraction, and therefore less backwater rise.

Figures 21, 22, and 23 show design curves based on the data from figures 18, 19, and 20. The data points and design curves have been presented on separate graphs for clarity. The curves were fit by taking into account not only the data points pertaining directly to each line, but also the relationship between the lines. The points labelled  $b/B = 0.50$  include values from 0.48 to 0.52;  $b/B = .625$  includes values from 0.605 to 0.645; and  $b/B = 0.75$  includes values between 0.73 and 0.77.

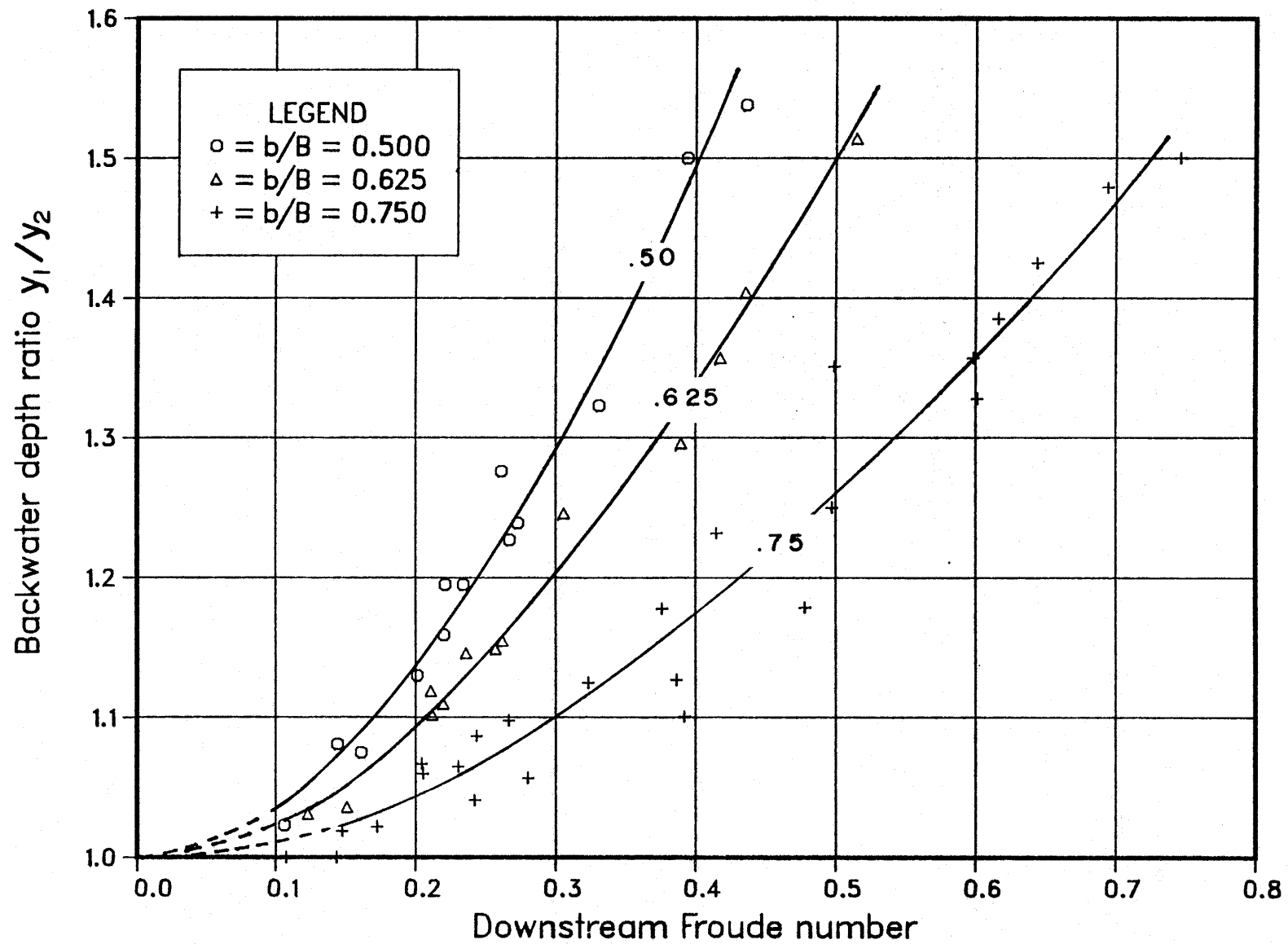


Fig. 17. Effect of contraction ratio on backwater depth for  $y_1/P < 1.05$ ; numbers on the lines are values of  $b/B$ .

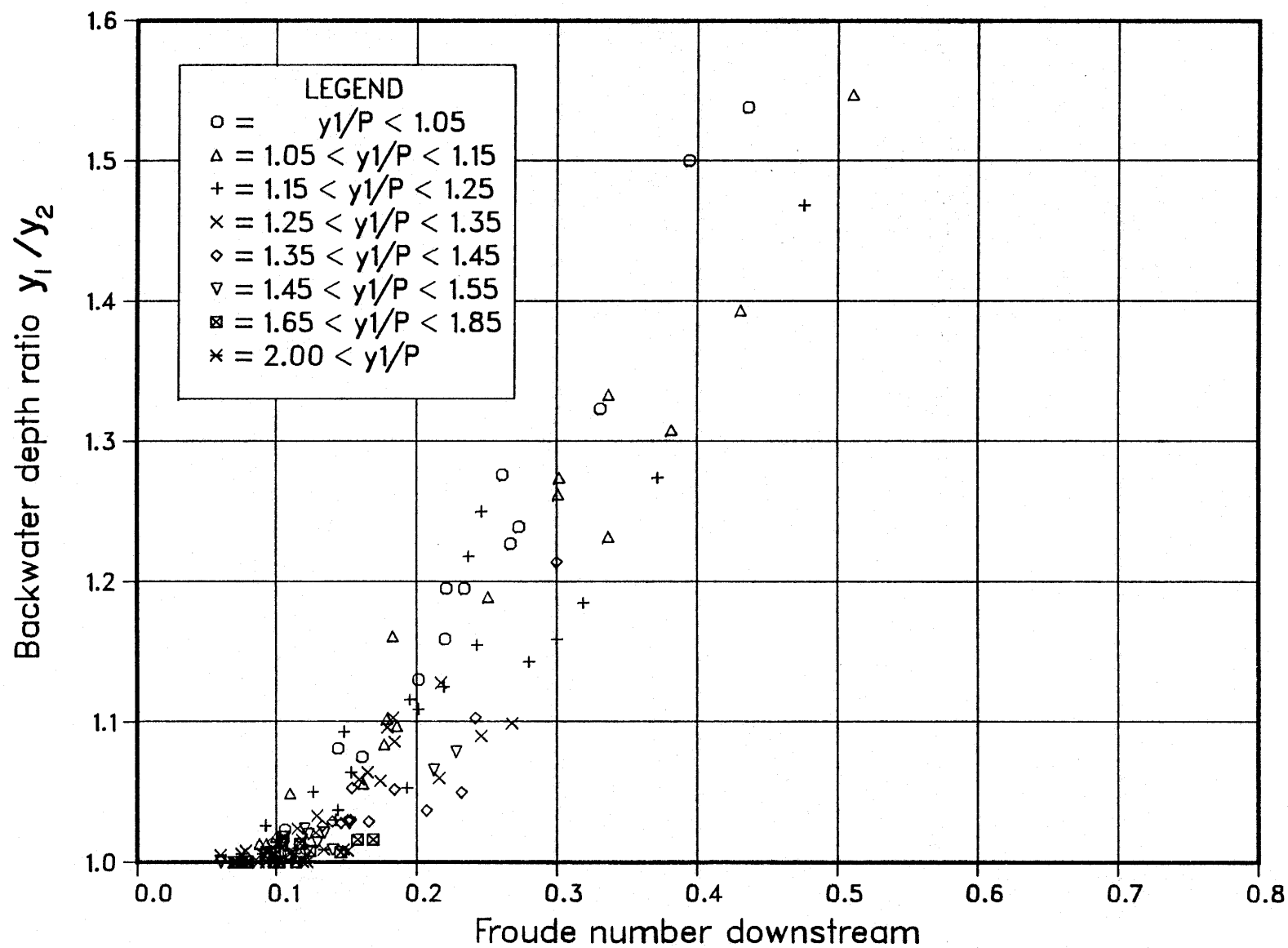


Fig. 18. Effect of submerged spur on backwater depth for  $b/B = 0.50$ .

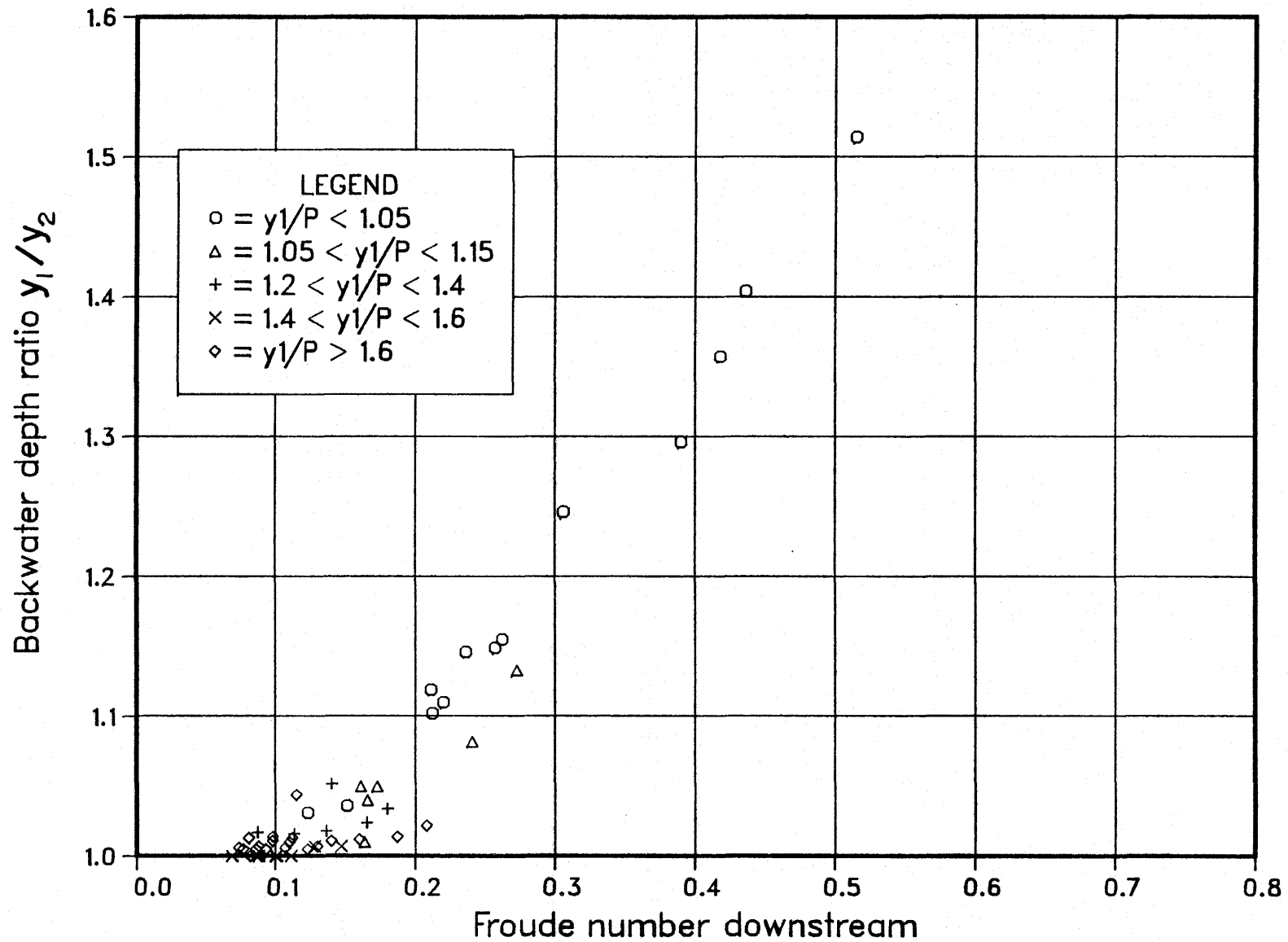


Fig. 19. Effect of submerged spur on backwater depth for  $b/B = 0.625$ .

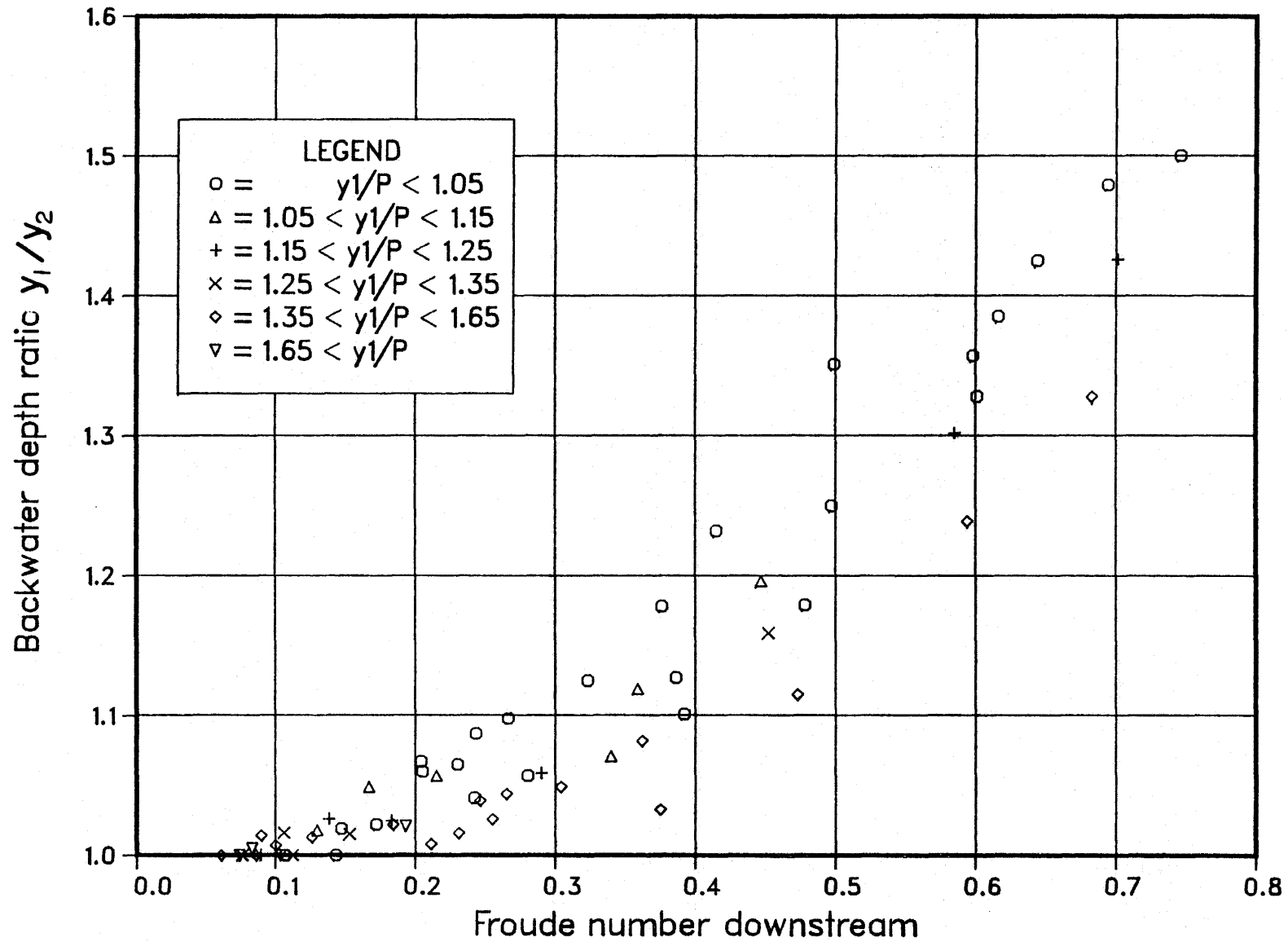


Fig. 20. Effect of submerged spur on backwater depth for  $b/B = 0.75$ .



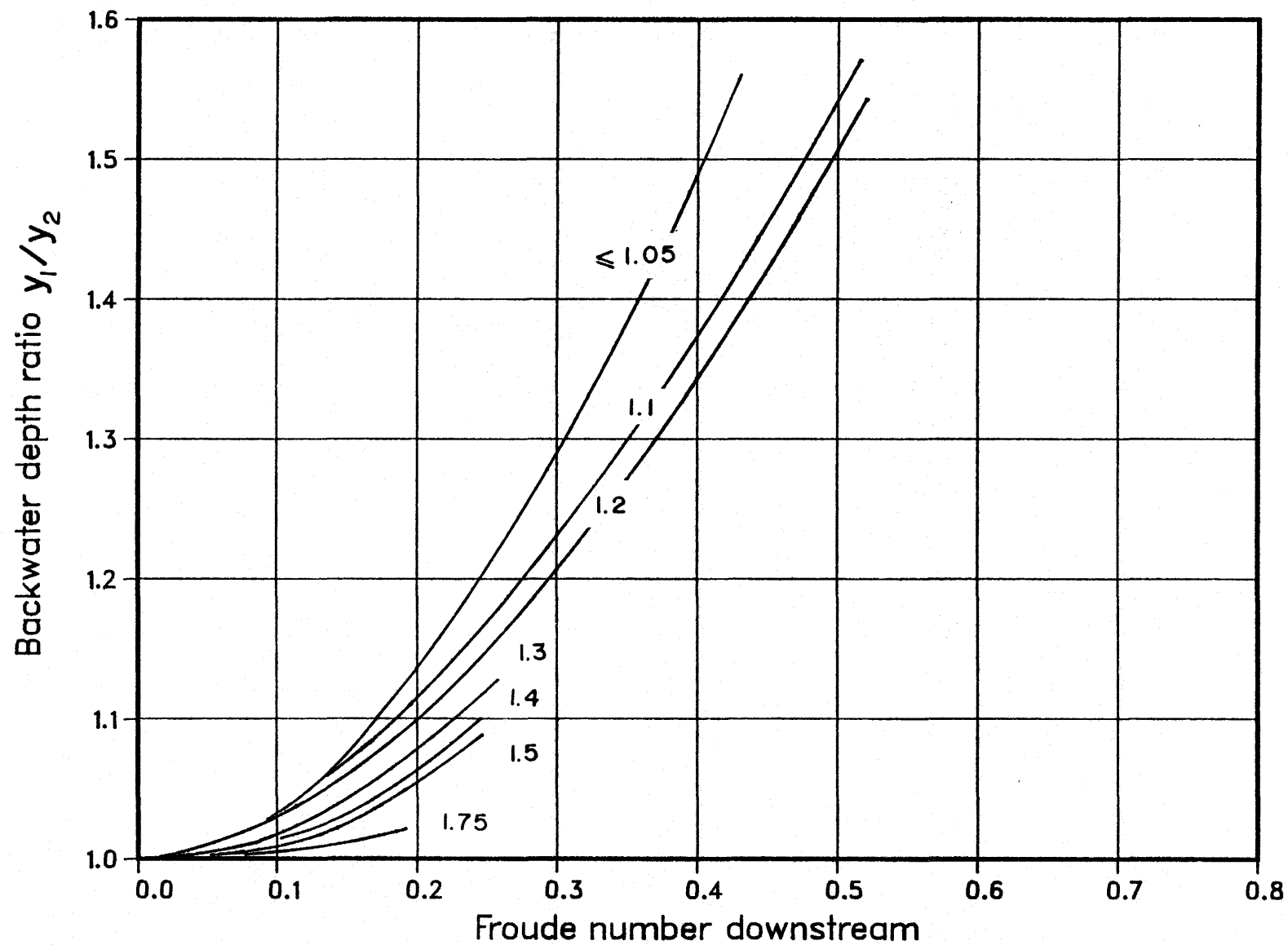


Fig. 21. Design curves for backwater for  $b/B = 0.50$ . Numbers on the lines are values of  $y_1/P$ .

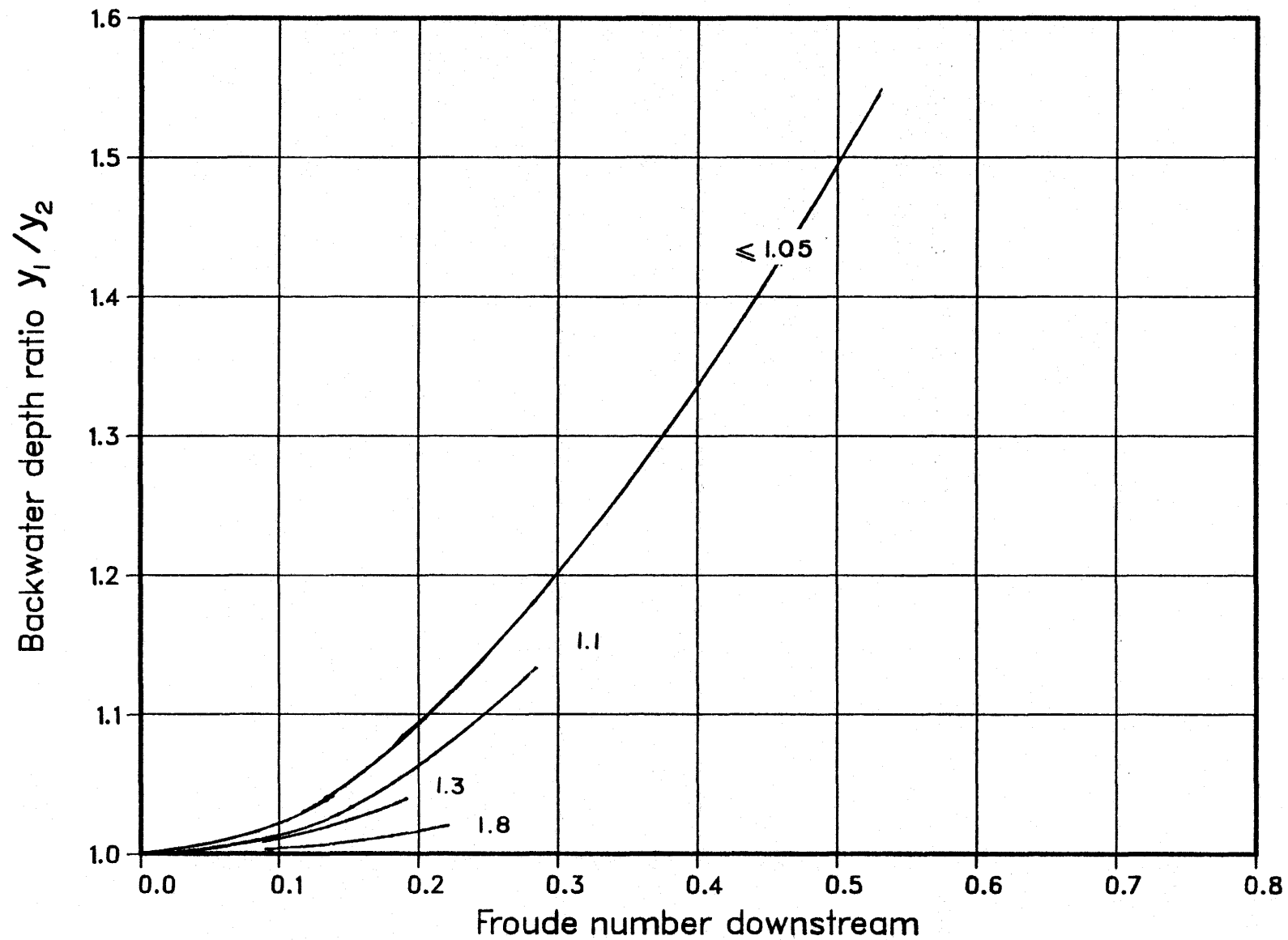


Fig. 22. Design curves for backwater for  $b/B = 0.625$ . Numbers on the lines are values of  $y_1/P$ .

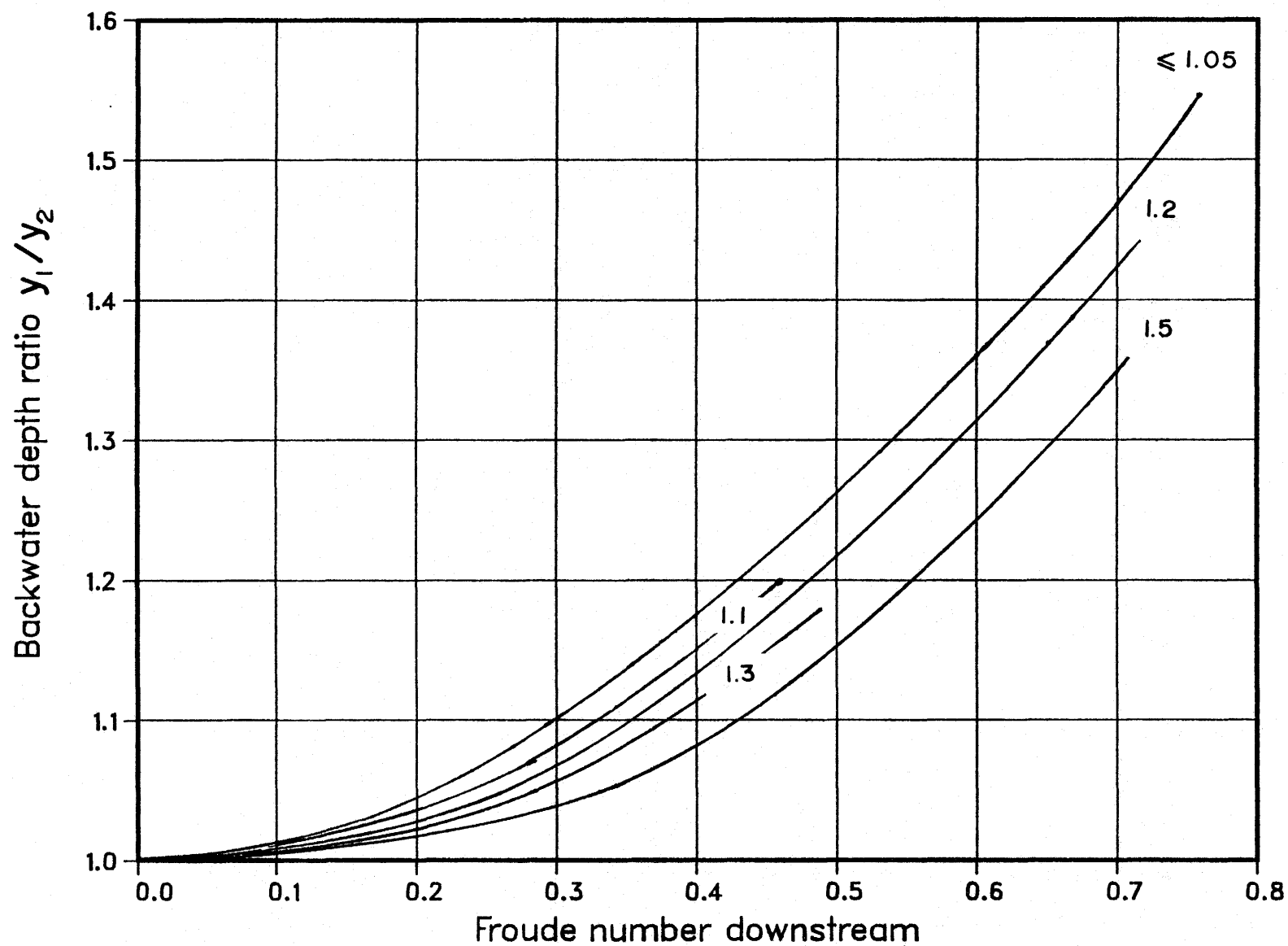


Fig. 23. Design curves for backwater for  $b/B = 0.75$ . Numbers on the lines are values of  $y_1/P$ .

An attempt was made to find a simple mathematical relationship which would describe the design curves. However, a simple computer curve fitting program could not take into account the data for points above and below those pertaining directly to the curve. The hand-drawn curves resulted in a more consistent family of curves, although the fit of each individual curve may not have been as good.

The value used for  $b$  in the analysis was the average width of the water cross section beyond the end of the spur crest, as shown in Fig. 24. In each case,  $b$  is the area of the shaded region divided by the depth  $y_1$ . The actual depth of water at the spur is not exactly  $y_1$ , but for the purpose of calculating the width this error was deemed to be negligible. The width recorded for each test in Appendix C is the width of the opening at the mid-height of the spur.

Figures 21, 22, and 23 may be used to find the depth upstream of a spur if the opening ratio, spur height, discharge, and downstream depths are known. The solution involves trial and error since initially the value of  $y_1/P$  is unknown, but usually only one or two iterations will be required. The method is to calculate the Froude number downstream, and then to guess a value for  $y_1/P$  based on a knowledge of  $y_2/P$ . Next the back-water ratio is found from the graph and  $y_1$  is solved. Then  $y_1/P$  is calculated and compared with the guessed

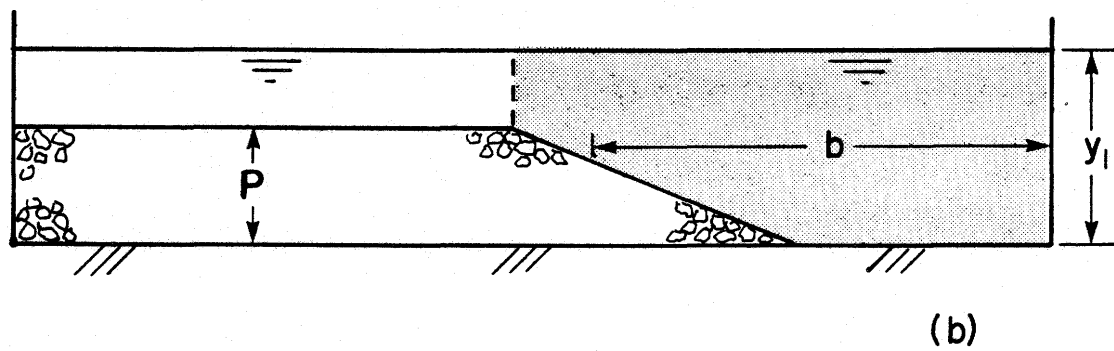
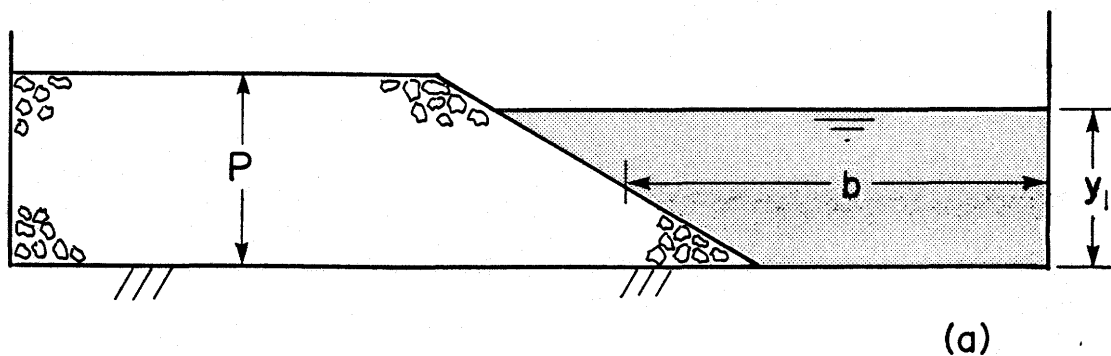


Fig. 24. Width of opening  $b$ . (a)  $y_1 < P$ ; (b)  $y_1 > P$ .

value, and the process is repeated if the values do not agree. An example of this procedure is given in Chapter 6.

#### E. Discharge Coefficients

Valentine (1958), using the discharge equation

$$Q = C b y_i^{1.5} \quad \dots(21)$$

showed that  $C$  is a function of the contraction ratio and the downstream Froude number. This was for rectangular restricting plates on an rigid bed. Das and Nuttall (1973) developed similar curves for an end-tipped embankment on an alluvial bed. In this case, the corresponding coefficients were much larger.

Fig. 25 shows the results of the work of Valentine and Das and Nuttall. Since equation (19) is not dimensionally correct, the value of the coefficient depends on the system of measurement used. The original work was done using imperial units. An ordinate with the corresponding SI coefficients has been added to allow comparison with the present work.

As part of the attempt to partition the flow into 'over' and 'around' components, discharge coefficients were developed for the cases in this study in which the upstream depth was not greater than the spur height. The discharge through the spur was assumed to be negli-

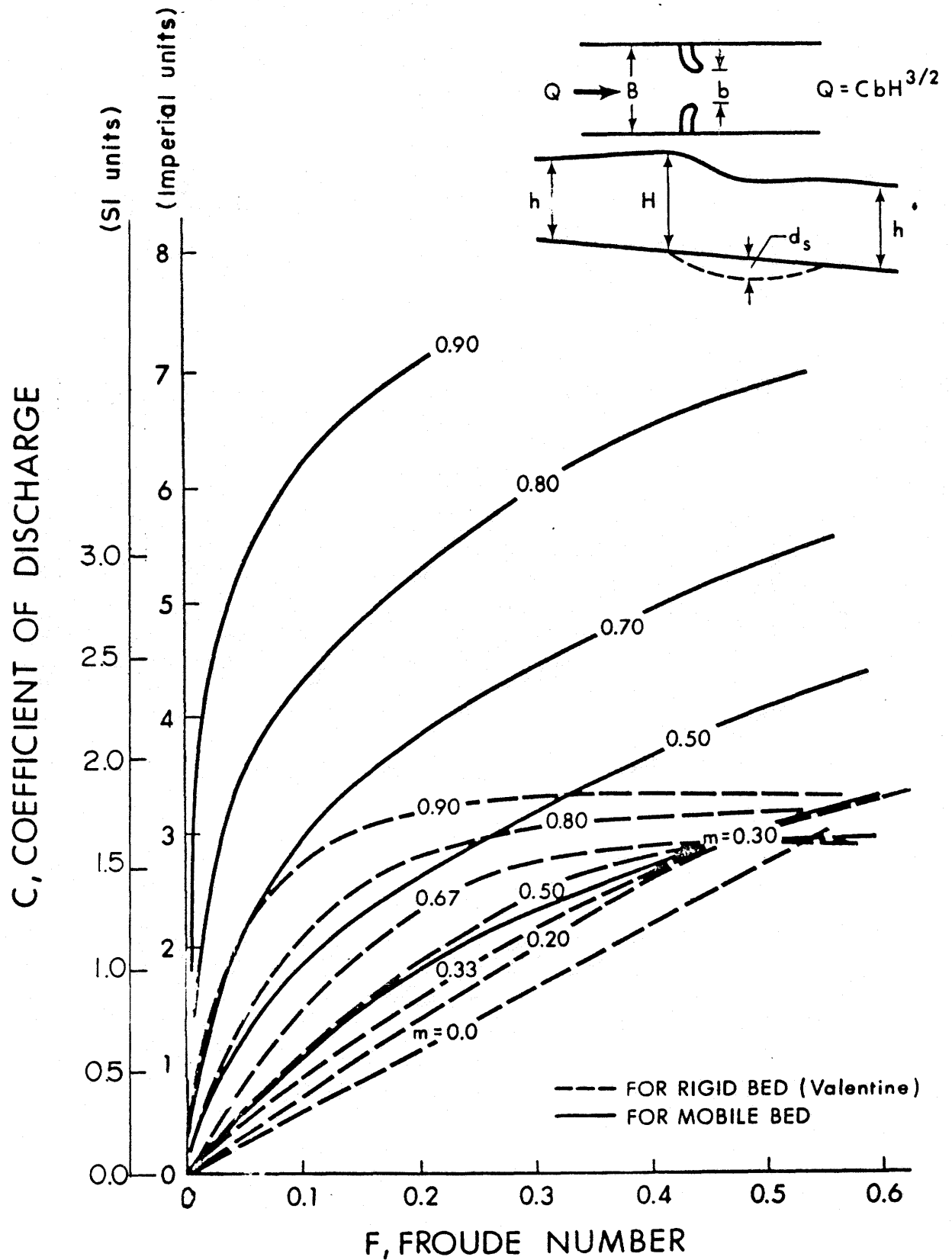


Fig. 25. Discharge coefficients for flow through a constriction. From Das and Nuttal, 1973.



gible. There is a good relationship between the discharge coefficient and the downstream Froude number (Fig. 26).

The simple nature of equation (21) requires the coefficient to include a variety of effects. Thus  $C$  must depend on, or take into account: the velocity of approach; the coefficient of lateral contraction; the coefficient of velocity; and the change of area of the flow due to scour in the contraction. It is this latter effect which accounts for the difference between the curves in Fig. 25. To compensate for the area of flow being larger than equation (21) indicates, the coefficient must be smaller.

Comparison of Fig. 26 with Fig. 25 shows that the results of the present study are very similar to those of Valentine, who used rectangular restricting plates rather than rock models. Evidently the coefficient is much more sensitive to the difference between an alluvial and a rigid bed than to a change in the geometry of the constricting element.

This relationship was not used either to partition the flow or to determine the depth ratio  $y_1/y_2$  because: (1) it was not possible to develop a simple function describing the weir coefficient for the overflow part of the discharge; and (2) the curve is very steep in the practical range of Froude numbers 0.1 to 0.2,

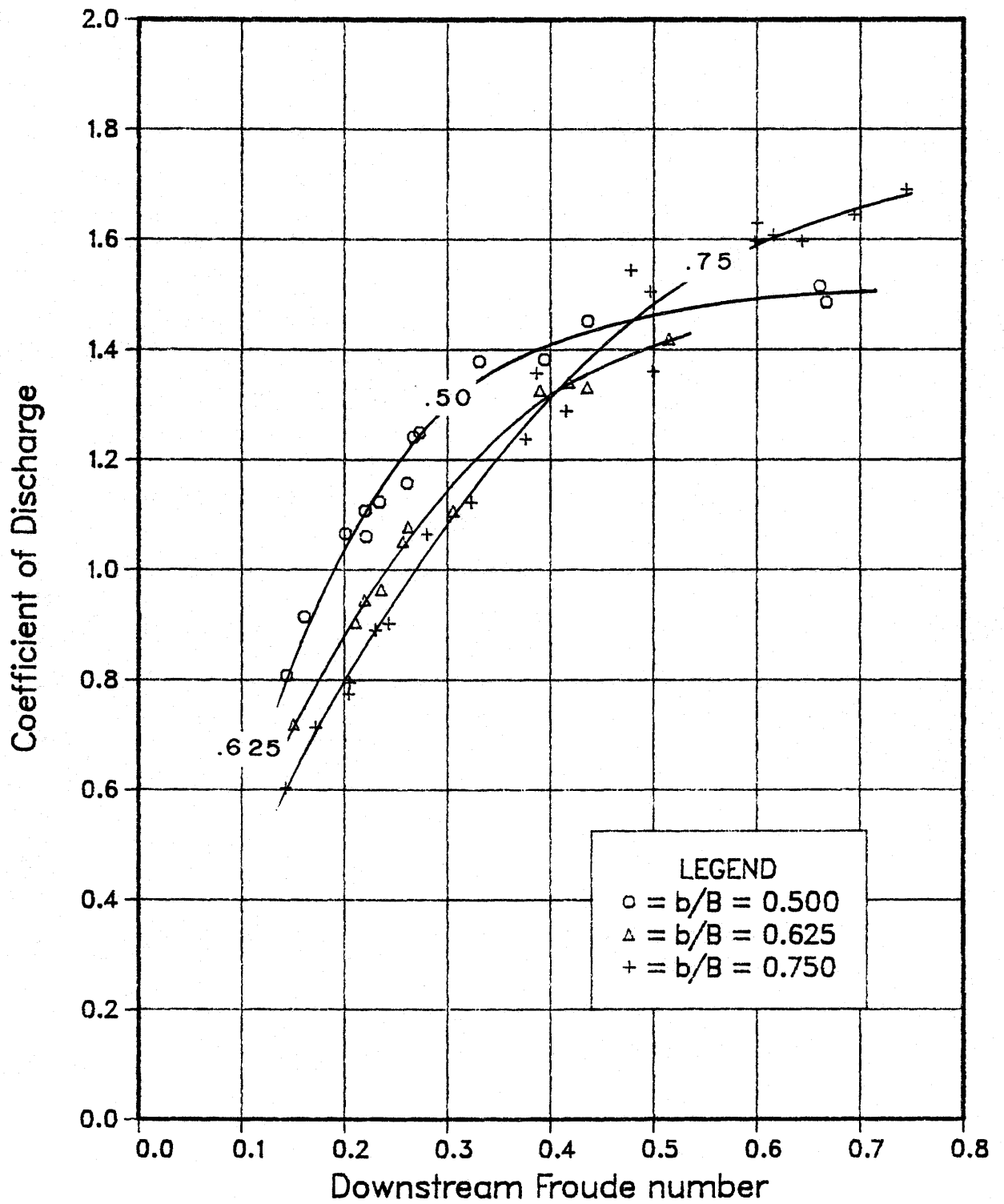


Fig. 26. Discharge coefficients for flow around the end of the spur only. Numbers on the lines are values of  $b/B$ .

leading to the possibility of large errors in the calculation of the discharge or the depth.

## CHAPTER 5

### CONFIRMATION TESTS

#### A. Rationale

The third set of tests was required to check that the results of the first set of tests (that the difference in water levels across the structure at first movement is 1.5 rock diameters) are true even when there is some flow around the end of the spur. The three-dimensional tests showed a different flow pattern just downstream of the spur; this could affect the stability of the rock on the crest.

Since from the definition sketch

$$h - h_s = y_1 - y_2 \quad \dots(22)$$

equation (15) can be written as

$$d_m = \frac{y_1 - y_2}{1.5} \quad \dots(23)$$

at first movement. If equation (23) is rearranged it becomes

$$\frac{y_1 - y_2}{d_m} = 1.5 \quad \dots(24)$$

The objective of the third set of tests, then, was to find whether first movement really did occur at values of  $(y_1 - y_2)/d_m$  near 1.5. Movement had been noted on several three-dimensional tests in the 800 mm flume; these results were analyzed and five further tests were done in the 2.5 m flume.

## B. Procedure

The procedure for these tests was the same as for the other three-dimensional tests. The difference was that in this case, smaller stone was used and first movement was observed. The crests of the spurs in the 2.5 m flume were spray-painted before the start of each test to make movement more obvious. All of the three-dimensional tests could be considered as one set; some of them were analyzed only for hydraulic performance, and others for both hydraulic performance and rock stability.

## C. Results

Detailed results of the tests are contained in Appendix B. Actual values of  $(y_1 - y_2) / d_m$  ranged from 0.90 to 1.58 (Table 2). Perhaps the reason for the average value being less than 1.5 is that the three-dimensional tests were done in the wider flume using longer spur lengths. With a greater length of spur exposed to the flow, there is a higher probability of some rock being aligned just right for it to move early.

The mode of "failure" was observed to be the same as in the two-dimensional tests. Movement occurred first at the downstream edge of the crest. Rock

Table 2

Difference in water levels at first movement

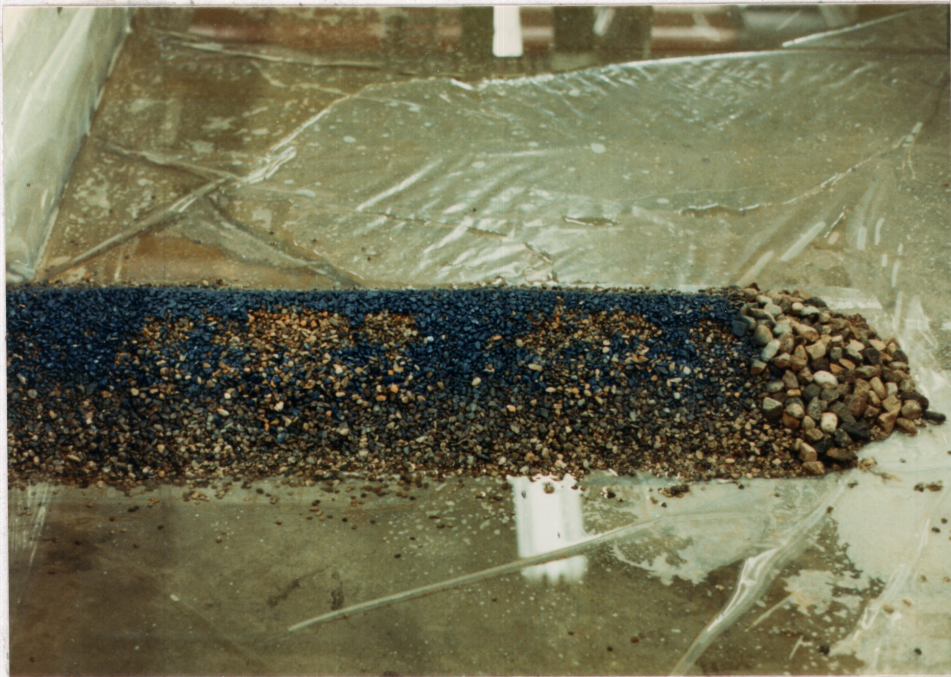
Flume width	Average	$(y_1 - y_2)/d_m$ Std. Dev.	Range
0.80 m	1.21	0.25	0.90 - 1.58
2.50 m	1.36	0.17	1.11 - 1.54
combined	1.26	0.23	0.90 - 1.58

dislodged from this area would tumble a short distance down the downstream slope before coming to rest. Even with the relatively steep downstream slope of 0.50 used for these tests, erosion of the downstream face was not observed. This suggests that it is unnecessary to flatten the downstream slope, as recommended by the Alberta Department of the Environment (1975). They suggest the use of downstream side slopes in the range of 0.33 to 0.20, and recommend flattening the slope even further if frequent overtopping is expected.

Of course, first movement does not necessarily constitute failure. "Failure" in the prototype might have a variety of definitions depending on the purpose of the spur, ease of maintenance, and other factors.

To see how "first movement" is related to failure, one test was continued well beyond first movement. First movement occurred in this case at  $(y_1 - y_2)/d_m = 1.11$ . At the end of the test,  $(y_1 - y_2)/d_m = 5$ . Even at this large differential head, only the downstream half of the crest was eroded, and for only about 50% of the length of the spur (Fig. 27). Evidently, numbers larger than 1.5 could be used for design, despite the fact that first movement often occurs at smaller numbers, if small amounts of movement are tolerable.





(a)



(b)

Fig. 27. Extent of erosion of crest at  $(y_1 - y_2)/d_m = 5$ . The crest and part of the downstream slope were spray-painted before the test to make observation of movement easier. (a) View from downstream; (b) end view. Note that the rock is deposited on the downstream slope of the spur, not carried away.



## CHAPTER 6

### DESIGN PROCEDURE

A design procedure for the spur crest stone size has been developed based on the results of this work. This procedure is given below, and an example is given to illustrate the method.

#### A. Design Procedure

##### 1. Gather the required field and design information.

The required information for the design of the rock size on the crest of the spur includes: the width of the stream  $B$ , the height of the spur  $P$ , the width of the opening  $b$ , the stage-discharge relationship for the stream before the spur is installed, and if possible, the frequency-discharge relationship.

##### 2. Calculate the downstream Froude numbers for various stages.

##### 3. Find the corresponding upstream depth $y_1$ for each stage using Figures 21, 22, or 23.

##### 4. Choose a value of $(y_1 - y_2)/d_m$ .

This would probably be between 1 and 5 and depends on the amount of risk the designer wants to assume. A larger number involves a higher risk.

Considerations in the choice of a number include: (1) the expected frequency of the critical discharge; (2) the consequences of a small amount of movement; (3) available stone sizes; (4) economy of frequent maintenance versus the use of a larger size of stone to begin with; and (5) the composition of the spur. A rockfill spur could tolerate more movement than could an earthfill spur topped with rock because removal of the rock layer on an earthfill spur could initiate catastrophic erosion of the spur material.

Considering that a value of 1.5 was found for first movement and 2.8 for group movement; and noting the extent of erosion for a value of 5 in Fig. 27; values of  $(y_1 - y_2)/d_m$  of 1.5 for rock covered earth fill and 2.5 for rock fill are recommended for a typical installation.

5. Calculate the design rock size.

Knowing  $y_2$  from step 1,  $y_1$  from step 3, and  $(y_1 - y_2)/d_m$  from step 4, calculate the required median rock diameter  $d_m$ .

The rock size should strictly be computed by this method only in the case of a prototype similar in all respects to the models on which the results are based; that is, a straight rockfill spur projecting perpendic-

ularly into a stream with a rectangular cross section and a uniform velocity distribution in plan upstream. The effect of small changes in these conditions is unknown. However, since the component of flow through the rockfill is relatively small, the results should apply quite well to an earthfill spur which has a layer of rock on the crest. Such a layer should be at least  $1.5 d_m$  thick to insure overlapping of the stone and eliminate the possibility of a void extending through the whole layer (Smith, 1978).

If the rock fill is graded rather than uniform, a slightly smaller stone size would be allowed since the effects of less porosity, greater interlocking (and so higher shear strength), and sheltering compensate for the smaller resistance to movement of the small stone (Smith, 1978, p. 194).

Perhaps the most serious limitation is the requirement for a uniform velocity distribution (in plan view) upstream of the spur. This is a rare condition in the field, and observations during the tests indicated that a non-uniform distribution has a large effect on both the head loss or backwater ratio caused by the spur and the stability of the rock on the crest.

In the Canadian climate, the design of the rock size on the crest may also have to take into account the possibility of ice. If the spur is likely to be

slightly submerged during ice breakup, the rock size required to resist movement by moving ice floes may well be much larger than would be required for stability under overtopping by water alone.

#### B. Example of a Design

In 1982, Saskatchewan Agriculture constructed a spur on the South Saskatchewan River at Chesterfield Flats near the Alberta - Saskatchewan border. The purpose of the spur was to prevent sediment deposition in front of a pump intake for an irrigation project. This spur was built of clay with a compacted layer of gravel on the crest. In the spring flood of 1984, the crest was eroded in several places. A layer of rock was placed on the crest to prevent a recurrence. The selection of this rock size will be done as an example of the design method.

Data obtained from the consultant (Smith, 1985) included a cross section of the river, the stage - discharge relationship, and a plan and profile of the spur. A summary of the relevant data is given in the first three columns of Table 3. The spur is not completely straight; it is L shaped with a very small leg projecting upstream. The main length of the spur is perpendicular to the bank.

Table 3Calculation of rock size for spur at Chesterfield Flats

Q	W.s. El.	B	b/B	Y <sub>2</sub>	F	Y <sub>1</sub>	Y <sub>1</sub> -Y <sub>2</sub>	d <sub>m</sub>
m <sup>3</sup> /s	m	m		m		m	m	m
150	573.95	345	0.507	1.55	0.111	1.60	0.05	0.033
200	574.20	345	0.507	1.80	0.138	1.91	0.11	0.073
250	574.45	345.5	0.507	2.05	0.161	2.19	0.14	0.09
350	574.80	350	0.500	2.40	0.206	2.62	0.22	0.15
450	575.10	352	0.497	2.70	0.248	2.94	0.24	0.16
550	575.30	354	0.494	2.90	0.291	3.13	0.23	0.15
650	575.50	355	0.493	3.10	0.332	--	--	--

From the plans, the following information is obtained: Spur crest elevation: 574.4 m; Average bed elevation: 572.4 m; width of opening at the end of the spur: 175 m over the entire range of depths shown in Table 3; side slopes of the embankment: 2 horizontal to 1 vertical, upstream and downstream; and crest length (parallel to the flow): 6 m. For this solution, the effect of the head of the spur is assumed to be negligible.

Using  $(y_1 - y_2)/d_m = 1.5$ , the solution is shown in Table 3. For example, consider the calculation for  $Q = 250 \text{ m}^3/\text{s}$ . From the given information, it is known that  $y_2 = 2.05 \text{ m}$  and  $B = 345.5 \text{ m}$ . The Froude number is computed to be 0.16. Assuming  $y_1/P$  to be 1.05, the value for  $y_1/y_2$  from Fig. 21 is 1.08. Then  $y_1 = 1.08 \times 2.05 \text{ m} = 2.21 \text{ m}$ . But now  $y_1/P = 2.21 / 2.0 = 1.1$ . Using Fig. 21 again for  $F_2 = 0.16$  and  $y_1/P = 1.1$ , then  $y_1/y_2 = 1.07$  and  $y_1 = 2.19 \text{ m}$ .

Now  $y_1 - y_2 = 2.19 \text{ m} - 2.05 \text{ m} = 0.14 \text{ m}$ . The rock size required for this flow condition is  $0.14 \text{ m} / 1.5 = 0.09 \text{ m}$ .

This calculation must be repeated for different discharges until the critical one is found. For this example, the curves for large submergences in Fig. 21 are not continued far enough to allow the completion of the calculation for  $Q = 650 \text{ m}^3/\text{s}$ . However, from the trend of the data, it appears that  $Q = 450 \text{ m}^3/\text{s}$  was the critical case.

From this information, a rock size of 160 mm median diameter would be recommended. In fact, the size used was between 200 and 300 mm. There has been one spring flood since this riprap was installed, and it has performed satisfactorily.

## CHAPTER 7

### CONCLUSION

#### A. SUMMARY

Model rockfill spurs were tested for the stability of the stone on the crest under the following conditions:

1. Subrounded to subangular stone, Specific Gravity about 2.7.
2. Uniform gradation of stone.
3. Straight spur with no head, level crest.
4. Spur projecting perpendicularly into the flow.
5. Stream with a rectangular cross-section.
6. Uniform velocity distribution in plan upstream of the spur.

Two-dimensional stability tests showed that first movement of the stone on the crest of a spur could be defined by the relationship

$$d_m = \frac{h - h_s}{1.5} \quad \dots(15)$$

For group movement the corresponding denominator was 2.8.

Three-dimensional tests were done to study the hydraulics of the flow over and around the spur, and to check the results of the two-dimensional tests. The backwater effect of the constriction was examined.



A design procedure was developed, based on the results of these tests.

Considerations in the application of these results should include the following:

1. Backwater across a spur located on an alluvial bed, in which scour occurs in the contraction, will be much less than on a rigid bed as was used in the test program.
2. A non-uniform lateral velocity distribution upstream of the spur was not accounted for in this study.

The first item above makes the results conservative in any case. The second item would make the results of the design unconservative if the higher velocity is directed against the spur.

## B. CONCLUSIONS

1. The Froude number downstream of a constriction is a useful independent variable, at least for the computation of backwater. Calculations based on it have fairly good accuracy. In contrast to the Froude number of the contracted section, the downstream Froude number is easily determined at the design stage.

2. The depth of water upstream of a constriction may be determined using the depth ratio  $y_1/y_2$ . The constriction may consist of a spur extending from one bank or of embankments extending from each bank, and may include overtopping of the constricting elements. For a given geometry, the depth ratio is a function only of the contraction ratio, the downstream Froude number, and the ratio of upstream depth to spur height (Fig. 21 - 23).
3. The coefficient of discharge for flow through a constriction for the case of no overtopping is also a function of the downstream Froude number and the contraction ratio (Fig. 26).
4. In the absence of any modifying factors, the rock size required for stability on the crest of a spur similar to the models used in this work is the difference in water depths across the spur divided by a constant. This constant is not dependent on the crest length, spur height, or downstream slope of the spur, but may vary with different velocity distributions upstream and different spur geometries than those tested.

### C. RECOMMENDATIONS

1. The recommended value for the constant mentioned in conclusion four is 1.5 for rock covered earth fill and 2.5 for a rockfill spur.
2. Experiments are required to find the backwater depth ratio for situations not covered by Figures 21 - 23. These include  $y_1/P$  ratios greater than about 1.3 for Froude numbers in the range of 0.2 to 0.5 and the contraction ratios shown; different contraction ratios; and possibly different spur geometries.

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## APPENDIX A

### LIST OF SYMBOLS

## LIST OF SYMBOLS

$b$	width of opening around the end of the spur
$B$	width of stream channel
$C_c$	Coefficient of lateral contraction
$C_D$	Coefficient of discharge
$d_m$	Median rock diameter
$d_2$	Minimum depth of the contracted jet
$F$	Froude number
$F_2$	Froude number applying to the downstream (uniform) flow
$g$	Acceleration of gravity
$h$	Depth of water above the crest upstream
$\Delta h$	Difference between upstream and minimum depths
$h_f$	Friction loss in the reach
$h_s$	Depth of water above the crest downstream
$h_{v1}$	Upstream velocity head
$h_1^*$	Backwater rise
$L$	Crest length parallel to the flow
$m$	Contraction ratio; $1 - b/B$
$n$	Manning's roughness parameter
$P$	Spur height
$q$	Discharge per unit width
$Q$	Discharge
$r$	Radius of curvature of the corners of the embankment
$S$	Slope of the downstream side of the embankment
$S_g$	Specific gravity of the stone
$v_1$	Upstream velocity

$v_2$	Downstream velocity
$y_1$	Upstream depth
$y_2$	Downstream depth
$y_n$	Normal depth
$\nu$	Fluid kinematic viscosity
$\rho$	Fluid density
$\rho_s$	Stone density

## APPENDIX B

### DATA FROM SET 1

## DATA FROM SET 1

The results of the first set of tests are tabulated below. The letters in the run number indicate which condition of movement has just occurred: FM means first movement, SM means second movement, and GM is group movement. Not all the conditions were noted for all the tests. When second movement and group movement were simultaneous, the resulting data was considered to be for the case of group movement.

Runs 2 through 78 were done in the 305 mm flume; 79 through 91 were in the 800 mm flume.



TABLE 4. RESULTS OF FIRST SET OF TESTS

RUN	Q m <sup>3</sup> /s	h cm	d <sub>m</sub> /h	P/h	L/h	S	h <sub>s</sub> /h
2-FM	0.01304	9.54	0.075	1.72	0.52	0.500	0.914
2-GM	0.01304	8.11	0.088	2.02	0.56	0.500	0.778
3-FM	0.01321	9.08	0.079	1.83	0.50	0.333	0.896
3-SM	0.01321	8.56	0.083	1.94	0.53	0.333	0.851
3-GM	0.01321	8.14	0.088	2.04	0.55	0.333	0.790
4-FM	0.01275	9.91	0.072	1.69	3.03	0.500	0.938
4-SM	0.01275	9.11	0.078	1.83	3.29	0.500	0.903
4-GM	0.01275	8.38	0.085	1.99	3.58	0.500	0.691
5-FM	0.00951	6.98	0.102	2.39	4.30	0.500	0.803
5-GM	0.00951	6.80	0.105	2.46	4.41	0.500	0.587
6-FM	0.01297	9.08	0.079	1.97	3.30	0.500	0.869
6-SM	0.01297	8.53	0.084	2.10	3.52	0.500	0.796
6-GM	0.01297	8.38	0.085	2.13	3.58	0.500	0.745
7-FM	0.00757	6.34	0.113	2.88	3.94	0.500	0.813
7-SM	0.00757	6.13	0.117	2.99	4.08	0.500	0.706
7-GM	0.00757	6.00	0.119	3.05	4.16	0.500	0.614
8-FM	0.00697	5.33	0.134	3.45	0.37	0.500	0.806
8-GM	0.00697	4.63	0.154	3.97	0.43	0.500	0.658
9-FM	0.00535	4.24	0.169	4.19	0.94	0.500	0.612
9-GM	0.00535	4.05	0.176	4.38	0.99	0.500	0.436
10-FM	0.01224	8.87	0.080	2.01	0.34	0.200	0.897
10-SM	0.01224	8.63	0.083	2.06	0.35	0.200	0.887
10-GM	0.01224	7.86	0.091	2.26	0.38	0.200	0.818
11-FM	0.00754	5.94	0.120	3.02	0.34	0.200	0.846
11-SM	0.00754	5.79	0.123	3.10	0.35	0.200	0.826
11-GM	0.00754	5.21	0.137	3.44	0.38	0.200	0.602
12-FM	0.01124	8.05	0.138	2.08	3.11	0.500	0.780
12-SM	0.01124	7.68	0.145	2.18	3.25	0.500	0.694
12-GM	0.01124	7.65	0.145	2.19	3.27	0.500	0.570
13-FM	0.00735	5.21	0.213	3.44	4.80	0.500	0.626
13-GM	0.00735	4.97	0.223	3.61	5.03	0.500	0.362
14-FM	0.01211	8.47	0.131	1.99	0.24	0.200	0.871
14-SM	0.01211	8.17	0.136	2.06	0.24	0.200	0.851
14-GM	0.01211	6.77	0.164	2.49	0.30	0.200	0.554
15-FM	0.00697	5.00	0.222	3.33	0.40	0.200	0.799
15-SM	0.00697	4.45	0.249	3.74	0.45	0.200	0.616
15-GM	0.00697	3.99	0.278	4.17	0.50	0.200	0.092
16-FM	0.00674	5.15	0.215	3.09	0.39	0.500	0.775
16-SM	0.00674	4.82	0.230	3.31	0.42	0.500	0.627
16-GM	0.00674	4.30	0.258	3.71	0.47	0.500	0.411
17-FM	0.01323	8.81	0.126	1.88	0.23	0.500	0.865
17-GM	0.01323	7.13	0.156	2.32	0.28	0.500	0.632
18-FM	0.00717	4.57	0.243	3.60	0.44	0.500	0.453
18-SM	0.00717	4.57	0.243	3.60	0.44	0.500	0.453
18-GM	0.00717	4.42	0.251	3.72	0.45	0.500	0.359
19-FM	0.00710	5.49	0.202	2.26	0.36	0.500	0.844
19-SM	0.00710	5.18	0.214	2.39	0.39	0.500	0.776

RUN	Q m <sup>3</sup> /s	h cm	d <sub>m</sub> /h	P/h	L/h	S	h <sub>s</sub> /h
19-GM	0.00710	4.79	0.232	2.59	0.42	0.500	0.637
20-FM	0.00657	4.97	0.223	4.42	0.40	0.500	0.785
20-SM	0.00657	4.57	0.243	4.81	0.44	0.500	0.713
20-GM	0.00657	4.21	0.264	5.22	0.48	0.500	0.623
21-FM	0.01230	8.56	0.130	2.54	0.23	0.500	0.861
21-SM	0.01230	8.08	0.137	2.70	0.25	0.500	0.819
21-GM	0.01230	7.38	0.150	2.95	0.27	0.500	0.719
22-FM	0.01288	7.47	0.213	2.19	0.40	0.500	0.706
22-SM	0.01288	7.47	0.213	2.19	0.40	0.500	0.706
22-GM	0.01288	7.47	0.213	2.19	0.40	0.500	0.706
23-FM	0.01298	9.42	0.169	1.83	0.32	0.500	0.922
23-SM	0.01298	7.41	0.215	2.33	0.41	0.500	0.782
23-GM	0.01298	6.71	0.237	2.58	0.45	0.500	0.641
24-FM	0.01076	6.55	0.243	2.71	0.46	0.500	0.763
24-GM	0.01076	6.04	0.263	2.94	0.50	0.500	0.586
25-FM	0.01221	7.32	0.217	2.29	0.41	0.500	0.683
25-GM	0.01221	6.95	0.229	2.41	0.43	0.500	0.566
26-FM	0.00812	6.80	0.105	2.26	0.29	0.500	0.901
26-GM	0.00812	6.19	0.115	2.49	0.32	0.500	0.847
27-FM	0.00814	6.04	0.118	2.47	0.33	0.500	0.823
27-GM	0.00814	5.67	0.126	2.63	0.35	0.500	0.731
28-FM	0.00874	7.62	0.094	1.95	0.26	0.500	0.924
28-SM	0.00874	6.92	0.103	2.15	0.29	0.500	0.881
28-GM	0.00874	6.40	0.112	2.32	0.31	0.500	0.790
29-FM	0.00834	6.61	0.108	2.24	0.30	0.500	0.876
29-GM	0.00834	6.04	0.118	2.45	0.33	0.500	0.758
30-FM	0.00684	5.30	0.135	2.09	0.38	0.500	0.799
30-GM	0.00684	4.97	0.144	2.23	0.40	0.500	0.730
31-FM	0.00667	5.09	0.140	1.99	0.39	0.500	0.796
31-GM	0.00667	4.88	0.146	2.07	0.41	0.500	0.744
32-FM	0.00674	5.82	0.123	1.75	0.34	0.500	0.880
32-SM	0.00674	5.36	0.133	1.90	0.37	0.500	0.835
32-GM	0.00674	5.03	0.142	2.02	0.40	0.500	0.745
33-FM	0.00671	5.43	0.132	2.10	0.37	0.500	0.854
33-GM	0.00671	4.97	0.144	2.29	0.40	0.500	0.767
34-FM	0.00732	5.49	0.130	3.14	0.36	0.500	0.794
34-GM	0.00732	5.49	0.130	3.14	0.36	0.500	0.794
35-FM	0.00748	6.40	0.112	2.72	1.25	0.500	0.876
35-SM	0.00748	5.76	0.124	3.02	1.39	0.500	0.757
35-GM	0.00748	5.58	0.128	3.13	1.43	0.500	0.421
36-FM	0.00757	5.97	0.120	2.85	2.01	0.500	0.791
36-SM	0.00757	5.79	0.123	2.94	2.07	0.500	0.732
36-GM	0.00757	5.64	0.127	3.02	2.13	0.500	0.686
37-FM	0.00763	6.22	0.115	2.68	1.61	0.500	0.848
37-SM	0.00763	5.82	0.123	2.86	1.72	0.500	0.738
37-GM	0.00763	5.67	0.126	2.94	1.76	0.500	0.651
38-FM	0.00766	6.31	0.113	2.78	3.65	0.500	0.816
38-SM	0.00766	6.16	0.116	2.85	3.74	0.500	0.802
38-GM	0.00766	5.94	0.120	2.95	3.87	0.500	0.672
39-FM	0.00763	6.13	0.117	2.83	3.75	0.500	0.756

RUN	$Q$ $m^3/s$	$h$ cm	$d_m/h$	$P/h$	$L/h$	$S$	$h_s/h$
39-SM	0.00763	6.04	0.118	2.87	3.81	0.500	0.707
39-GM	0.00763	5.94	0.120	2.91	3.87	0.500	0.605
40-FM	0.00744	5.97	0.120	2.96	5.36	0.500	0.730
40-SM	0.00744	5.91	0.121	2.99	5.41	0.500	0.701
40-GM	0.00744	5.82	0.123	3.04	5.50	0.500	0.602
41-FM	0.00754	6.25	0.114	2.79	4.80	0.500	0.780
41-SM	0.00754	6.10	0.117	2.85	4.92	0.500	0.665
41-GM	0.00754	6.04	0.118	2.88	4.97	0.500	0.551
42-FM	0.00741	6.28	0.114	2.79	0.32	0.200	0.947
42-SM	0.00741	6.00	0.119	2.91	0.33	0.200	0.883
42-GM	0.00741	5.43	0.132	3.22	0.37	0.200	0.803
43-FM	0.00757	6.46	0.110	1.55	11.30	0.200	0.835
43-SM	0.00757	6.37	0.112	1.57	11.46	0.200	0.766
43-GM	0.00757	6.31	0.113	1.59	11.57	0.200	0.667
44-FM	0.00754	6.00	0.119	1.67	4.50	0.200	0.792
44-SM	0.00754	5.94	0.120	1.69	4.54	0.200	0.764
44-GM	0.00754	5.91	0.121	1.70	4.57	0.200	0.701
45-FM	0.00754	6.31	0.113	1.63	4.28	0.500	0.845
45-SM	0.00754	6.13	0.117	1.68	4.41	0.500	0.801
45-GM	0.00754	5.91	0.121	1.74	4.57	0.500	0.634
46-FM	0.00763	6.19	0.115	1.59	3.23	0.500	0.847
46-SM	0.00763	6.07	0.118	1.62	3.30	0.500	0.819
46-GM	0.00763	5.94	0.120	1.65	3.36	0.500	0.697
47-FM	0.00763	5.91	0.121	1.69	3.38	0.200	0.835
47-SM	0.00763	5.79	0.123	1.72	3.45	0.200	0.716
47-GM	0.00763	5.76	0.124	1.73	3.47	0.200	0.646
48-FM	0.00757	6.28	0.114	1.53	1.91	0.200	0.898
48-SM	0.00757	6.04	0.118	1.59	1.99	0.200	0.833
48-GM	0.00757	5.97	0.120	1.61	2.01	0.200	0.806
49-FM	0.00751	6.22	0.115	1.65	0.32	0.200	0.926
49-SM	0.00751	5.94	0.120	1.72	0.34	0.200	0.903
49-GM	0.00751	5.43	0.132	1.89	0.37	0.200	0.798
50-FM	0.00741	6.07	0.118	1.62	3.46	0.500	0.799
50-SM	0.00741	6.04	0.118	1.63	3.48	0.500	0.747
50-GM	0.00741	5.91	0.121	1.66	3.55	0.500	0.634
51-FM	0.00754	6.10	0.117	1.63	1.80	0.500	0.825
51-SM	0.00754	6.04	0.118	1.65	1.82	0.500	0.788
51-GM	0.00754	5.88	0.121	1.69	1.87	0.500	0.715
52-FM	0.00766	6.00	0.119	1.73	1.00	0.500	0.858
52-SM	0.00766	5.79	0.123	1.79	1.04	0.500	0.826
52-GM	0.00766	5.58	0.128	1.86	1.08	0.500	0.781
53-FM	0.00766	6.22	0.115	1.55	0.56	0.500	0.863
53-SM	0.00766	5.76	0.124	1.68	0.61	0.500	0.794
53-GM	0.00766	5.58	0.128	1.73	0.63	0.500	0.732
54-FM	0.00760	6.19	0.115	2.78	0.48	0.500	0.867
54-SM	0.00760	5.61	0.127	3.07	0.53	0.500	0.766
54-GM	0.00760	5.52	0.129	3.12	0.54	0.500	0.751
55-FM	0.00751	6.34	0.113	2.79	0.32	0.500	0.880
55-SM	0.00751	5.67	0.126	3.12	0.35	0.500	0.817
55-GM	0.00751	5.00	0.143	3.54	0.40	0.500	0.659

RUN	$Q$ $m^3/s$	$h$ cm	$d_m/h$	$P/h$	$L/h$	$S$	$h_s/h$
56-FM	0.00780	6.22	0.115	2.88	0.32	0.500	0.858
56-SM	0.00780	5.76	0.124	3.11	0.35	0.500	0.788
56-GM	0.00780	5.76	0.124	3.11	0.35	0.500	0.788
57-FM	0.00771	5.82	0.123	2.85	1.20	0.500	0.780
57-SM	0.00771	5.70	0.125	2.91	1.23	0.500	0.727
57-GM	0.00771	5.55	0.129	2.99	1.26	0.500	0.670
58-FM	0.00760	6.22	0.115	2.70	1.13	0.500	0.863
58-SM	0.00760	5.73	0.125	2.93	1.22	0.500	0.766
58-GM	0.00760	5.52	0.129	3.04	1.27	0.500	0.630
59-FM	0.00757	5.97	0.120	2.89	1.51	0.500	0.791
59-SM	0.00757	5.76	0.124	2.99	1.56	0.500	0.746
59-GM	0.00757	5.61	0.127	3.08	1.60	0.500	0.668
60-FM	0.00748	5.76	0.124	2.96	0.17	0.200	0.847
60-SM	0.00748	5.46	0.131	3.13	0.18	0.200	0.782
60-GM	0.00748	5.46	0.131	3.13	0.18	0.200	0.782
61-FM	0.00766	5.61	0.127	3.13	0.36	0.200	0.793
61-SM	0.00766	5.43	0.132	3.24	0.37	0.200	0.770
61-GM	0.00766	5.12	0.139	3.43	0.39	0.200	0.595
62-FM	0.00763	6.00	0.119	2.92	0.33	0.200	0.868
62-SM	0.00763	5.58	0.128	3.14	0.36	0.200	0.803
62-GM	0.00763	5.46	0.131	3.21	0.37	0.200	0.760
63-FM	0.00754	4.94	0.322	2.57	0.61	0.500	0.679
63-SM	0.00754	4.75	0.334	2.67	0.63	0.500	0.590
63-GM	0.00754	4.24	0.375	3.00	0.71	0.500	0.108
64-FM	0.00760	4.57	0.348	3.39	0.66	0.500	0.187
64-SM	0.00760	4.57	0.348	3.39	0.66	0.500	0.187
64-GM	0.00760	4.57	0.348	3.39	0.66	0.500	0.187
65-FM	0.00604	3.38	0.470	4.46	0.89	0.500	0.171
65-SM	0.00604	3.32	0.479	4.54	0.90	0.500	0.018
65-GM	0.00604	3.11	0.511	4.85	0.96	0.500	-0.480
66-FM	0.00607	4.27	0.373	3.59	0.70	0.500	0.514
66-SM	0.00607	3.90	0.408	3.93	0.77	0.500	0.078
66-GM	0.00607	3.87	0.411	3.96	0.78	0.500	0.000
67-FM	0.00576	4.30	0.370	3.85	3.96	0.500	0.348
67-SM	0.00576	4.18	0.381	3.96	4.07	0.500	0.124
67-GM	0.00576	4.02	0.395	4.11	4.23	0.500	-0.220
68-FM	0.00798	4.91	0.324	3.97	0.61	0.500	0.584
68-SM	0.00798	4.21	0.378	4.63	0.71	0.500	0.036
68-GM	0.00798	4.21	0.378	4.63	0.71	0.500	0.036
69-FM	0.00591	3.66	0.435	6.18	1.37	0.500	0.400
69-SM	0.00591	3.38	0.470	6.68	1.48	0.500	0.090
69-GM	0.00591	3.08	0.516	7.34	1.62	0.500	-0.554
70-FM	0.00751	4.05	0.392	5.66	1.23	0.500	0.271
70-SM	0.00751	3.90	0.408	5.88	1.28	0.500	0.008
70-GM	0.00751	3.08	0.516	7.46	1.62	0.500	-0.307
71-FM	0.01083	5.97	0.266	3.86	0.50	0.500	0.592
71-SM	0.01083	5.67	0.280	4.06	0.53	0.500	0.446
71-GM	0.01083	5.18	0.307	4.45	0.58	0.500	0.071
72-FM	0.00828	5.06	0.314	4.49	0.59	0.500	0.651
72-SM	0.00828	4.69	0.339	4.84	0.64	0.500	0.442

RUN	Q m <sup>3</sup> /s	h cm	d <sub>m</sub> /h	P/h	L/h	S	h <sub>s</sub> /h
72-GM	0.00828	4.24	0.375	5.37	0.71	0.500	0.108
73-FM	0.00834	4.66	0.341	4.78	0.64	0.500	0.307
73-SM	0.00834	4.57	0.348	4.87	0.66	0.500	0.153
73-GM	0.00834	4.42	0.360	5.04	0.68	0.500	-0.028
74-FM	0.00828	4.79	0.332	4.68	0.63	0.500	0.414
74-SM	0.00828	4.63	0.343	4.83	0.65	0.500	0.276
74-GM	0.00828	4.27	0.373	5.24	0.70	0.500	-0.364
75-FM	0.00457	2.65	0.600	8.36	1.13	0.500	-0.218
75-SM	0.00457	2.47	0.644	8.98	1.22	0.500	-0.617
75-GM	0.00457	1.98	0.803	11.18	1.51	0.500	-3.400
76-FM	0.00522	3.26	0.488	6.68	0.92	0.500	0.262
76-SM	0.00522	2.87	0.555	7.61	1.05	0.500	-0.543
76-GM	0.00509	2.59	0.614	8.41	1.16	0.500	-1.388
77-FM	0.00504	3.11	0.511	7.06	1.61	0.500	0.353
77-SM	0.00504	2.93	0.543	7.50	1.71	0.500	0.094
77-GM	0.00504	2.65	0.600	8.28	1.89	0.500	-0.448
78-FM	0.00424	2.41	0.660	8.90	2.08	0.500	0.253
78-SM	0.00424	2.50	0.636	8.57	2.00	0.500	-0.037
78-GM	0.00424	1.83	0.869	11.72	2.73	0.500	-5.300
79-FM	0.01049	1.68	0.948	14.27	2.98	0.500	-0.673
79-SM	0.01049	1.89	0.841	12.66	2.65	0.500	-0.806
79-GM	0.01049	0.67	2.371	35.68	7.46	0.500	-13.136
80-FM	0.01393	2.62	0.607	9.16	1.53	0.500	0.209
80-SM	0.01393	2.50	0.636	9.61	1.60	0.500	0.061
80-GM	0.01393	2.38	0.669	10.10	1.68	0.500	-0.244
81-FM	0.01171	2.44	0.652	9.47	1.64	0.500	0.225
81-SM	0.01171	2.32	0.686	9.97	1.73	0.500	0.013
81-GM	0.01171	1.86	0.855	12.43	2.15	0.500	-1.328
82-FM	0.01128	2.29	0.696	9.97	2.19	0.500	-0.080
82-SM	0.01128	2.04	0.779	11.16	2.45	0.500	-0.567
82-GM	0.01128	1.83	0.869	12.47	2.73	0.500	-1.533
83-FM	0.01116	2.80	0.567	8.14	1.78	0.500	0.391
83-SM	0.01116	2.59	0.614	8.81	1.93	0.500	0.294
83-GM	0.01116	2.07	0.767	11.01	2.41	0.500	-0.941
84-FM	0.01093	2.26	0.705	11.09	1.33	0.500	0.230
84-SM	0.01093	2.01	0.790	12.44	1.49	0.500	0.000
84-GM	0.01093	4.66	0.341	5.37	0.64	0.500	-0.379
85-FM	0.01049	2.01	0.790	12.45	2.49	0.500	0.015
85-SM	0.01049	1.77	0.899	14.17	2.83	0.500	-0.345
85-GM	0.01049	1.22	1.304	20.55	4.10	0.500	-3.425
86-FM	0.01059	1.71	0.932	14.87	2.93	0.500	-0.161
86-SM	0.01059	1.62	0.984	15.72	3.10	0.500	-0.434
86-GM	0.01059	0.73	2.174	34.71	6.84	0.500	-8.417
87-FM	0.01081	1.68	0.948	15.16	2.98	0.500	-0.418
87-SM	0.01081	1.52	1.043	16.68	3.28	0.500	-0.900
87-GM	0.01081	1.04	1.534	24.53	4.82	0.500	-2.824
88-FM	0.01083	1.68	0.948	15.02	2.39	0.500	-0.855
89-FM	0.01064	2.07	0.767	12.09	1.93	0.500	0.191
89-SM	0.01064	1.71	0.932	14.68	2.34	0.500	-0.589
89-GM	0.01064	1.46	1.087	17.12	2.73	0.500	-1.812

RUN	$Q$ $m^3/s$	$h$ cm	$d_m/h$	$P/h$	$L/h$	$S$	$h_s/h$
90-FM	0.01097	2.68	0.593	7.77	9.32	0.500	0.011
90-SM	0.01097	2.56	0.621	8.14	9.76	0.500	-0.214
90-GM	0.01097	2.44	0.652	8.55	10.25	0.500	-0.950
91-FM	0.01066	2.59	0.614	8.20	9.65	5.000	-0.012
91-SM	0.01066	2.41	0.660	8.82	10.38	5.000	-0.367
91-GM	0.01066	2.19	0.725	9.68	11.39	5.000	-1.375



## APPENDIX C

### DATA FROM THREE-DIMENSIONAL TESTS

## DATA FROM THREE-DIMENSIONAL TESTS

The data for the three-dimensional tests is listed below. Table 6 contains a summary of the important parameters for each test; Table 7 contains the detailed results. For each test: RUN is the run number, DM is the median rock diameter in m, Q is the discharge in  $\text{m}^3/\text{s}$ , P is the spur height in m, BA is the average width of the opening at the end of the spur in m, Z is the inverse of the slope of the end of the spur, B is the channel width, FRAV is a discharge parameter  $Q/(\sqrt{g}L^{2.5})$  using BA for the length dimension, FRTOP is the discharge parameter using the top width of the opening for the length dimension, PG1 and PG2 are the point gauge readings of the water levels upstream and downstream of the spur, and Y1 and Y2 are the corresponding water depths in m. An asterisk indicates the point at which movement of the crest was first observed.

Two flumes and several different point gauges were used for the tests as shown in Table 5.

Run 39 is omitted because partway through the test it was observed that the velocity distribution upstream of the spur was not uniform. This made the results of this run inconsistent with those of the other tests.

Table 5

Flume and point gauge data for three-dimensional tests

Runs	Point gauge units	Flume width	Bed Elevations		
			Upstream	Model	Downstream
1 - 6	cm	800 mm	49.87	49.82	49.83
7 - 8	cm	800 mm	47.28	47.28	47.22
9 - 24	tenths of feet	800 mm	3.80	3.80	3.80
25 - 43	feet	2.50 m	0.600	0.671	-0.129

On tests 38 through 43 the crest of the spur was spray-painted before the test, and the crest was carefully observed for movement. First movement was also noted for many of the tests in the 800 mm flume, but the crests had not been painted.

Table 6: Summary of Three-Dimensional Tests

RUN	DM m	Q m <sup>3</sup> /s	P m	BA m	Z	P/B	BA/B	FRAV
1	0.0111	0.0373	0.085	0.438	1.6	0.107	0.547	0.0941
2	0.0111	0.0265	0.085	0.438	1.6	0.107	0.547	0.0668
3	0.0111	0.0184	0.078	0.403	1.1	0.097	0.503	0.0572
4	0.0111	0.0256	0.082	0.567	1.4	0.102	0.709	0.0337
5	0.0111	0.0250	0.103	0.270	1.5	0.129	0.338	0.2107
6	0.0159	0.0254	0.109	0.268	1.4	0.136	0.334	0.2191
7	0.0159	0.0258	0.113	0.000	0.0	0.142	0.000	0.0000
8	0.0159	0.0180	0.113	0.000	0.0	0.141	0.000	0.0000
9	0.0159	0.0263	0.106	0.395	1.6	0.133	0.494	0.0856
10	0.0159	0.0177	0.116	0.383	1.9	0.145	0.478	0.0625
11	0.0159	0.0260	0.213	0.385	1.9	0.267	0.481	0.0903
12	0.0159	0.0250	0.089	0.310	2.0	0.112	0.387	0.1492
13	0.0159	0.0187	0.088	0.310	2.1	0.110	0.387	0.1116
14	0.0159	0.0362	0.093	0.325	2.3	0.116	0.406	0.1920
15	0.0159	0.0517	0.092	0.320	2.0	0.115	0.400	0.2850
16	0.0222	0.0180	0.070	0.440	2.6	0.087	0.550	0.0448
17	0.0222	0.0127	0.070	0.440	2.6	0.088	0.550	0.0316
18	0.0222	0.0259	0.070	0.440	2.6	0.088	0.550	0.0644
19	0.0222	0.0122	0.102	0.590	1.6	0.128	0.738	0.0146
20	0.0222	0.0183	0.097	0.610	1.7	0.121	0.762	0.0201
21	0.0222	0.0261	0.097	0.610	1.7	0.121	0.762	0.0287
22	0.0222	0.0129	0.058	0.595	1.9	0.072	0.744	0.0151
23	0.0222	0.0179	0.058	0.595	1.9	0.072	0.744	0.0209
24	0.0222	0.0258	0.058	0.595	1.9	0.072	0.744	0.0302

RUN	DM m	Q m <sup>3</sup> /s	P m	BA m	Z	P/B	BA/B	FRAV
25	0.0159	0.0584	0.089	1.430	3.4	0.035	0.572	0.0076
26	0.0159	0.0574	0.095	1.250	2.3	0.038	0.500	0.0105
27	0.0159	0.0405	0.095	1.250	2.3	0.038	0.500	0.0074
28	0.0159	0.0405	0.095	1.565	2.0	0.038	0.626	0.0042
29	0.0159	0.0287	0.095	1.565	2.0	0.038	0.626	0.0030
30	0.0159	0.0578	0.095	1.565	2.0	0.038	0.626	0.0060
31	0.0159	0.0434	0.095	1.565	2.0	0.038	0.626	0.0045
32	0.0159	0.0278	0.100	1.875	1.9	0.040	0.750	0.0018
33	0.0159	0.0431	0.100	1.875	1.9	0.040	0.750	0.0029
34	0.0159	0.0581	0.100	1.875	1.9	0.040	0.750	0.0039
35	0.0159	0.0285	0.100	1.250	2.0	0.040	0.500	0.0052
36	0.0159	0.0430	0.100	1.250	2.0	0.040	0.500	0.0079
37	0.0159	0.0575	0.100	1.250	2.0	0.040	0.500	0.0105
38	0.0071	0.0406	0.100	1.250	2.0	0.040	0.500	0.0074
40	0.0071	0.0570	0.100	1.250	2.0	0.040	0.500	0.0104
41	0.0071	0.0405	0.082	1.250	1.8	0.033	0.500	0.0074
42	0.0071	0.0429	0.075	1.250	2.0	0.030	0.500	0.0078
43	0.0071	0.0572	0.075	1.250	2.0	0.030	0.500	0.0105

Table 7: Detailed Results of Three-Dimensional Tests.

RUN	DM	Q	P	BA	Z	
1	0.0111	0.0373	0.085	0.438	1.58	
	P/B	BA/B	FRAV	FRTOP		
	0.1065	0.547	0.0941	0.0657		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
74.64	74.48	0.248	0.247	2.907	2.893	0.11
72.90	72.82	0.230	0.230	2.703	2.698	0.04
70.84	70.76	0.210	0.209	2.461	2.457	0.04
68.74	68.58	0.189	0.188	2.215	2.201	0.11
66.62	66.30	0.168	0.165	1.966	1.933	0.25
65.80	65.42	0.159	0.156	1.870	1.830	0.31
64.78	64.28	0.149	0.144	1.750	1.696	0.41
63.52	62.72	0.137	0.129	1.602	1.513	0.68
63.20	62.08	0.133	0.123	1.565	1.438	0.97
62.80	61.22	0.129	0.114	1.518	1.337	1.39
62.50	60.52	0.126	0.107	1.482	1.255	1.75

RUN	DM	Q	P	BA	Z	
2	0.0111	0.0265	0.085	0.438	1.58	
	P/B	BA/B	FRAV	FRTOP		
	0.1065	0.547	0.0668	0.0467		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
67.46	67.34	0.176	0.175	2.065	2.055	0.07
66.74	66.60	0.169	0.168	1.980	1.968	0.09
65.96	65.94	0.161	0.161	1.888	1.891	-0.02
65.00	64.80	0.151	0.150	1.776	1.757	0.14
64.26	64.04	0.144	0.142	1.689	1.668	0.16
63.56	63.20	0.137	0.134	1.607	1.569	0.29
62.82	62.42	0.130	0.126	1.520	1.478	0.32
62.16	61.56	0.123	0.117	1.442	1.377	0.50
61.74	60.90	0.119	0.111	1.393	1.299	0.72
61.32	60.16	0.115	0.103	1.344	1.212	1.01 *
61.06	59.64	0.112	0.098	1.313	1.151	1.24
60.82	59.06	0.110	0.092	1.285	1.083	1.55
60.64	58.66	0.108	0.088	1.264	1.036	1.75
60.56	58.20	0.107	0.084	1.255	0.982	2.09
60.42	57.48	0.105	0.076	1.238	0.898	2.61
60.34	57.05	0.105	0.072	1.229	0.847	2.93
60.16	56.50	0.103	0.067	1.208	0.783	3.26
60.06	56.10	0.102	0.063	1.196	0.736	3.53



RUN	DM	Q	P	BA	Z	
3	0.0111	0.0184	0.078	0.403	1.09	
	P/B	BA/B	FRAV	FRTOP		
	0.0972	0.503	0.0572	0.0445		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
67.76	67.68	0.179	0.178	2.299	2.294	0.04
66.77	66.74	0.169	0.169	2.172	2.174	-0.01
65.80	65.72	0.159	0.159	2.048	2.042	0.04
64.88	64.80	0.150	0.150	1.929	1.924	0.04
63.96	63.80	0.141	0.140	1.811	1.796	0.11
63.20	63.07	0.133	0.132	1.713	1.702	0.08
62.48	62.26	0.126	0.124	1.621	1.598	0.16
61.74	61.43	0.119	0.116	1.526	1.491	0.24
61.10	60.64	0.112	0.108	1.443	1.389	0.38
60.40	59.82	0.105	0.100	1.353	1.284	0.49
59.90	58.90	0.100	0.091	1.289	1.166	0.86
59.46	57.98	0.096	0.081	1.233	1.048	1.30 *
59.12	57.14	0.093	0.073	1.189	0.940	1.75
58.94	56.02	0.091	0.062	1.166	0.796	2.59
58.86	55.32	0.090	0.055	1.156	0.706	3.15
58.72	54.54	0.089	0.047	1.138	0.605	3.73
58.64	54.40	0.088	0.046	1.127	0.587	3.78

RUN	DM	Q	P	BA	Z	
4	0.0111	0.0256	0.082	0.567	1.41	
	P/B	BA/B	FRAV	FRTOP		
	0.1023	0.709	0.0337	0.0265		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
64.30	63.92	0.144	0.141	1.764	1.722	0.31
63.30	63.14	0.134	0.133	1.642	1.627	0.11
62.54	62.32	0.127	0.125	1.549	1.527	0.16
61.82	61.50	0.120	0.117	1.461	1.427	0.25
61.08	60.62	0.112	0.108	1.370	1.319	0.38
60.42	59.74	0.105	0.099	1.290	1.211	0.58
59.84	58.84	0.100	0.090	1.219	1.101	0.86
59.26	57.88	0.094	0.080	1.148	0.984	1.21 *

RUN	DM	Q	P	BA	Z	
5	0.0111	0.0250	0.103	0.270	1.55	
	P/B	BA/B	FRAV	FRTOP		
	0.1292	0.338	0.2107	0.1101		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
70.64	70.54	0.208	0.207	2.009	2.003	0.05
69.72	69.56	0.199	0.197	1.920	1.908	0.11
68.82	68.60	0.190	0.188	1.833	1.815	0.16
67.82	67.66	0.180	0.178	1.736	1.724	0.11
67.02	66.66	0.171	0.168	1.659	1.628	0.29
66.08	65.74	0.162	0.159	1.568	1.539	0.27
65.26	64.66	0.154	0.148	1.488	1.434	0.50
64.48	63.64	0.146	0.138	1.413	1.336	0.72
63.94	62.70	0.141	0.129	1.361	1.245	1.08
63.62	62.04	0.138	0.122	1.330	1.181	1.39 *
63.32	61.28	0.135	0.114	1.301	1.107	1.80
62.96	60.66	0.131	0.108	1.266	1.047	2.04
62.74	59.92	0.129	0.101	1.245	0.976	2.50
62.52	59.26	0.127	0.094	1.223	0.912	2.90
62.16	58.40	0.123	0.086	1.189	0.829	3.35

RUN	DM	Q	P	BA	Z	
6	0.0159	0.0254	0.109	0.268	1.42	
	P/B	BA/B	FRAV	FRTOP		
	0.1360	0.334	0.2191	0.1160		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
73.74	73.56	0.239	0.237	2.194	2.181	0.09
71.30	71.15	0.214	0.213	1.970	1.960	0.07
70.36	70.18	0.205	0.203	1.883	1.870	0.09
69.28	69.00	0.194	0.192	1.784	1.762	0.15
68.32	68.06	0.185	0.182	1.696	1.676	0.14
67.40	67.10	0.175	0.173	1.611	1.587	0.16
66.52	66.12	0.166	0.163	1.530	1.497	0.23
65.64	65.12	0.158	0.153	1.449	1.405	0.30
64.86	64.10	0.150	0.143	1.378	1.312	0.45
64.16	63.00	0.143	0.132	1.313	1.210	0.70
63.58	61.82	0.137	0.120	1.260	1.102	1.08 *
63.24	61.14	0.134	0.113	1.229	1.040	1.30
63.08	60.74	0.132	0.109	1.214	1.003	1.45
62.82	60.02	0.130	0.102	1.190	0.937	1.74
62.64	59.26	0.128	0.094	1.174	0.867	2.10

RUN	DM	Q	P	BA	Z	
7	0.0159	0.0258	0.113	0.000	0.00	
	P/B	BA/B	FRAV	FRTOP		
	0.1418	0.000	0.0000	0.0000		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
71.62	71.48	0.243	0.243	2.146	2.139	0.05
70.74	70.50	0.235	0.233	2.069	2.053	0.11
69.90	69.58	0.226	0.224	1.995	1.972	0.16
68.90	68.70	0.216	0.215	1.907	1.894	0.09
68.22	67.82	0.209	0.206	1.847	1.817	0.21
67.28	66.90	0.200	0.197	1.764	1.735	0.20
66.46	65.92	0.192	0.187	1.691	1.649	0.30
65.72	64.96	0.184	0.177	1.626	1.564	0.44
65.10	63.98	0.178	0.168	1.571	1.478	0.67
64.62	63.00	0.173	0.158	1.529	1.392	0.98
64.26	62.00	0.170	0.148	1.497	1.303	1.38
63.64	61.06	0.164	0.138	1.443	1.220	1.58 *
63.30	60.46	0.160	0.132	1.413	1.168	1.75
63.14	59.64	0.159	0.124	1.399	1.095	2.16

RUN	DM	Q	P	BA	Z	
8	0.0159	0.0180	0.113	0.000	0.00	
	P/B	BA/B	FRAV	FRTOP		
	0.1408	0.000	0.0000	0.0000		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
68.00	67.82	0.207	0.206	1.840	1.829	0.08
67.08	66.84	0.198	0.196	1.758	1.742	0.11
66.20	65.90	0.189	0.187	1.680	1.659	0.15
65.34	64.96	0.181	0.177	1.604	1.575	0.20
64.54	63.96	0.173	0.167	1.533	1.487	0.33
63.82	63.08	0.165	0.159	1.469	1.409	0.43
63.28	62.10	0.160	0.149	1.421	1.321	0.70
63.10	61.52	0.158	0.143	1.405	1.270	0.96 *
62.90	60.94	0.156	0.137	1.387	1.218	1.19
62.74	60.38	0.155	0.132	1.373	1.169	1.45

RUN	DM	Q	P	BA	Z
9	0.0159	0.0263	0.106	0.395	1.60

P/B	BA/B	FRAV	FRTOP
0.1326	0.494	0.0856	0.0526

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
11.44	11.41	0.233	0.232	2.195	2.187	0.06
11.15	11.10	0.224	0.223	2.112	2.098	0.10
10.83	10.80	0.214	0.213	2.020	2.011	0.06
10.51	10.47	0.205	0.203	1.928	1.917	0.08
10.20	10.15	0.195	0.194	1.839	1.825	0.10
9.90	9.83	0.186	0.184	1.753	1.733	0.13
9.59	9.52	0.176	0.174	1.664	1.644	0.13
9.28	9.19	0.167	0.164	1.575	1.549	0.17
8.98	8.87	0.158	0.155	1.489	1.457	0.21
8.69	8.53	0.149	0.144	1.405	1.359	0.31
8.41	8.17	0.141	0.133	1.325	1.256	0.46
8.17	7.81	0.133	0.122	1.256	1.152	0.69
8.00	7.49	0.128	0.112	1.207	1.060	0.98
7.88	7.30	0.124	0.107	1.172	1.006	1.11 *
7.79	7.05	0.122	0.099	1.147	0.934	1.42
7.71	6.78	0.119	0.091	1.124	0.856	1.78
7.63	6.57	0.117	0.084	1.101	0.796	2.03
7.59	6.27	0.116	0.075	1.089	0.710	2.53

RUN	DM	Q	P	BA	Z
10	0.0159	0.0177	0.116	0.383	1.94

P/B	BA/B	FRAV	FRTOP
0.1448	0.478	0.0625	0.0328

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.20	10.16	0.195	0.194	1.684	1.674	0.08
9.87	9.82	0.185	0.183	1.597	1.584	0.10
9.58	9.50	0.176	0.174	1.521	1.500	0.15
9.27	9.20	0.167	0.165	1.439	1.421	0.13
8.96	8.88	0.157	0.155	1.358	1.337	0.15
8.66	8.56	0.148	0.145	1.279	1.253	0.19
8.37	8.22	0.139	0.135	1.203	1.163	0.29
8.10	7.88	0.131	0.124	1.132	1.074	0.42
7.86	7.51	0.124	0.113	1.068	0.976	0.67
7.62	7.19	0.116	0.103	1.005	0.892	0.82
7.41	6.81	0.110	0.092	0.950	0.792	1.15
7.22	6.40	0.104	0.079	0.900	0.684	1.57
7.11	5.94	0.101	0.065	0.871	0.563	2.24

RUN	DM	Q	P	BA	Z
11	0.0159	0.0260	0.213	0.385	1.92

P/B	BA/B	FRAV	FRTOP
0.2667	0.481	0.0903	0.0310

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
13.91	13.88	0.308	0.307	1.444	1.440	0.06
13.60	13.56	0.299	0.297	1.400	1.394	0.08
13.32	13.25	0.290	0.288	1.360	1.350	0.13
12.99	12.93	0.280	0.278	1.313	1.304	0.12
12.68	12.63	0.271	0.269	1.269	1.261	0.10
12.38	12.30	0.262	0.259	1.226	1.214	0.15
12.09	12.00	0.253	0.250	1.184	1.171	0.17
11.76	11.69	0.243	0.240	1.137	1.127	0.13
11.48	11.37	0.234	0.231	1.097	1.081	0.21
11.19	11.04	0.225	0.221	1.056	1.034	0.29
10.96	10.78	0.218	0.213	1.023	0.997	0.35
10.66	10.48	0.209	0.204	0.980	0.954	0.35
10.37	10.17	0.200	0.194	0.939	0.910	0.38
10.09	9.81	0.192	0.183	0.899	0.859	0.54
9.83	9.49	0.184	0.173	0.861	0.813	0.65
9.56	9.16	0.176	0.163	0.823	0.766	0.77
9.31	8.87	0.168	0.155	0.787	0.724	0.84
9.09	8.54	0.161	0.144	0.756	0.677	1.05
8.90	8.13	0.155	0.132	0.729	0.619	1.48
8.73	7.66	0.150	0.118	0.704	0.551	2.05
8.63	7.28	0.147	0.106	0.690	0.497	2.59
8.56	6.88	0.145	0.094	0.680	0.440	3.22

RUN	DM	Q	P	BA	Z
12	0.0159	0.0250	0.089	0.310	2.02

P/B	BA/B	FRAV	FRTOP
0.1116	0.387	0.1492	0.0789

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.90	10.85	0.216	0.215	2.423	2.406	0.10
10.58	10.54	0.207	0.205	2.314	2.300	0.08
10.26	10.20	0.197	0.195	2.205	2.184	0.12
9.90	9.85	0.186	0.184	2.082	2.065	0.10
9.60	9.51	0.177	0.174	1.980	1.949	0.17
9.28	9.20	0.167	0.165	1.870	1.843	0.15
8.98	8.88	0.158	0.155	1.768	1.734	0.19
8.68	8.54	0.149	0.144	1.666	1.618	0.27
8.38	8.20	0.140	0.134	1.563	1.502	0.35
8.11	7.85	0.131	0.123	1.471	1.382	0.50
7.90	7.48	0.125	0.112	1.399	1.256	0.81
7.71	7.11	0.119	0.101	1.334	1.130	1.15
7.63	6.70	0.117	0.088	1.307	0.990	1.78

RUN	DM	Q	P	BA	Z
13	0.0159	0.0187	0.088	0.310	2.05

P/B	BA/B	FRAV	FRTOP
0.1097	0.387	0.1116	0.0590

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.81	10.71	0.214	0.211	2.434	2.399	0.19
10.50	10.45	0.204	0.203	2.326	2.309	0.10
10.01	9.95	0.189	0.187	2.156	2.135	0.12
9.51	9.45	0.174	0.172	1.983	1.962	0.12
9.02	8.94	0.159	0.157	1.813	1.785	0.15
8.53	8.45	0.144	0.142	1.642	1.615	0.15
8.06	7.92	0.130	0.126	1.479	1.431	0.27
7.77	7.57	0.121	0.115	1.378	1.309	0.38
7.54	7.20	0.114	0.104	1.299	1.181	0.65
7.37	6.84	0.109	0.093	1.240	1.056	1.02
7.24	6.45	0.105	0.081	1.194	0.920	1.51 *

RUN	DM	Q	P	BA	Z
14	0.0159	0.0362	0.093	0.325	2.27

P/B	BA/B	FRAV	FRTOP
0.1158	0.406	0.1920	0.0953

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
12.32	12.27	0.260	0.258	2.803	2.786	0.10
11.81	11.76	0.244	0.243	2.635	2.618	0.10
11.31	11.25	0.229	0.227	2.470	2.451	0.12
10.81	10.74	0.214	0.212	2.306	2.283	0.13
10.37	10.29	0.200	0.198	2.161	2.135	0.15
9.87	9.76	0.185	0.182	1.997	1.961	0.21
9.57	9.42	0.176	0.171	1.898	1.849	0.29
9.25	9.08	0.166	0.161	1.793	1.737	0.33
8.97	8.72	0.158	0.150	1.701	1.618	0.48
8.71	8.36	0.150	0.139	1.615	1.500	0.67
8.50	8.00	0.143	0.128	1.546	1.382	0.96 *
8.34	7.58	0.138	0.115	1.493	1.243	1.46

RUN	DM	Q	P	BA	Z	
15	0.0159	0.0517	0.092	0.320	1.96	
	P/B	BA/B	FRAV	FRTOP		
	0.1151	0.400	0.2850	0.1534		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
13.93	13.88	0.309	0.307	3.354	3.338	0.10
13.42	13.38	0.293	0.292	3.185	3.172	0.08
12.93	12.87	0.278	0.276	3.023	3.003	0.12
12.42	12.34	0.263	0.260	2.854	2.828	0.15
12.10	12.02	0.253	0.251	2.748	2.722	0.15
11.79	11.69	0.244	0.240	2.646	2.613	0.19
11.48	11.37	0.234	0.231	2.543	2.507	0.21
11.15	11.05	0.224	0.221	2.434	2.401	0.19
10.85	10.71	0.215	0.211	2.334	2.288	0.27
10.53	10.39	0.205	0.201	2.228	2.182	0.27
10.21	10.02	0.195	0.190	2.123	2.060	0.36
9.85	9.60	0.184	0.177	2.003	1.921	0.48
9.57	9.23	0.176	0.166	1.911	1.798	0.65
9.32	8.85	0.168	0.154	1.828	1.672	0.90 *
9.10	8.46	0.162	0.142	1.755	1.543	1.23
8.92	8.03	0.156	0.129	1.695	1.401	1.71

RUN	DM	Q	P	BA	Z	
16	0.0222	0.0180	0.070	0.440	2.58	
	P/B	BA/B	FRAV	FRTOP		
	0.0872	0.550	0.0448	0.0281		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.60	10.56	0.207	0.206	2.969	2.952	0.05
10.10	10.05	0.192	0.191	2.751	2.729	0.07
9.60	9.55	0.177	0.175	2.533	2.511	0.07
9.10	9.05	0.162	0.160	2.314	2.293	0.07
8.60	8.55	0.146	0.145	2.096	2.074	0.07
8.10	8.03	0.131	0.129	1.878	1.847	0.10
7.63	7.52	0.117	0.113	1.672	1.624	0.15
7.32	7.19	0.107	0.103	1.537	1.480	0.18
7.04	6.82	0.099	0.092	1.415	1.319	0.30
6.89	6.71	0.094	0.089	1.349	1.271	0.25
6.77	6.42	0.091	0.080	1.297	1.144	0.48
6.66	6.13	0.087	0.071	1.249	1.017	0.73
6.58	5.84	0.085	0.062	1.214	0.891	1.02
6.52	5.59	0.083	0.055	1.188	0.782	1.28
6.50	5.36	0.082	0.048	1.179	0.681	1.57
6.49	5.24	0.082	0.044	1.175	0.629	1.72
6.49	5.20	0.082	0.043	1.175	0.611	1.77



RUN	DM	Q	P	BA	Z
17	0.0222	0.0127	0.070	0.440	2.57

P/B	BA/B	FRAV	FRTOP
0.0876	0.550	0.0316	0.0198

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
9.36	9.31	0.169	0.168	2.417	2.396	0.07
9.04	9.00	0.160	0.158	2.278	2.261	0.05
8.73	8.68	0.150	0.149	2.143	2.122	0.07
8.41	8.37	0.141	0.139	2.004	1.987	0.05
8.12	8.05	0.132	0.130	1.878	1.848	0.10
7.79	7.73	0.122	0.120	1.735	1.709	0.08
7.50	7.42	0.113	0.110	1.609	1.574	0.11
7.18	7.10	0.103	0.101	1.470	1.435	0.11
6.89	6.76	0.094	0.090	1.343	1.287	0.18
6.72	6.55	0.089	0.084	1.270	1.196	0.23
6.58	6.33	0.085	0.077	1.209	1.100	0.34
6.44	6.16	0.080	0.072	1.148	1.026	0.38
6.32	5.95	0.077	0.066	1.096	0.935	0.51
6.23	5.65	0.074	0.056	1.057	0.804	0.80
6.15	5.43	0.072	0.050	1.022	0.709	0.99
6.10	5.20	0.070	0.043	1.000	0.609	1.24
6.09	5.04	0.070	0.038	0.996	0.539	1.44
6.08	4.99	0.069	0.036	0.991	0.517	1.50

RUN	DM	Q	P	BA	Z
18	0.0222	0.0259	0.070	0.440	2.57

P/B	BA/B	FRAV	FRTOP
0.0876	0.550	0.0644	0.0404

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.65	10.60	0.209	0.207	2.978	2.957	0.07
10.14	10.09	0.193	0.192	2.757	2.735	0.07
9.64	9.59	0.178	0.176	2.539	2.517	0.07
9.14	9.08	0.163	0.161	2.322	2.296	0.08
8.63	8.55	0.147	0.145	2.100	2.065	0.11
8.33	8.22	0.138	0.135	1.970	1.922	0.15
8.14	8.00	0.132	0.128	1.887	1.826	0.19
7.97	7.82	0.127	0.123	1.813	1.748	0.21
7.80	7.61	0.122	0.116	1.739	1.657	0.26
7.63	7.38	0.117	0.109	1.665	1.557	0.34
7.46	7.18	0.112	0.103	1.591	1.470	0.38
7.31	6.95	0.107	0.096	1.526	1.370	0.49
7.20	6.72	0.104	0.089	1.478	1.270	0.66
7.10	6.42	0.101	0.080	1.435	1.139	0.93
7.03	6.16	0.098	0.072	1.404	1.026	1.19
7.01	5.88	0.098	0.063	1.396	0.904	1.55 *
6.94	5.64	0.096	0.056	1.365	0.800	1.78

RUN	DM	Q	P	BA	Z
19	0.0222	0.0122	0.102	0.590	1.57

P/B	BA/B	FRAV	FRTOP
0.1276	0.738	0.0146	0.0106

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.97	10.93	0.219	0.217	2.140	2.128	0.05
10.46	10.44	0.203	0.202	1.988	1.982	0.03
9.97	9.94	0.188	0.187	1.842	1.833	0.04
9.47	9.44	0.173	0.172	1.693	1.684	0.04
9.04	9.02	0.160	0.159	1.564	1.558	0.03
8.56	8.50	0.145	0.143	1.421	1.403	0.08
8.05	8.00	0.130	0.128	1.269	1.254	0.07
7.55	7.49	0.114	0.112	1.119	1.101	0.08
7.24	7.18	0.105	0.103	1.027	1.009	0.08
6.93	6.85	0.095	0.093	0.934	0.910	0.11
6.63	6.52	0.086	0.083	0.845	0.812	0.15
6.33	6.18	0.077	0.073	0.755	0.710	0.21
6.07	5.87	0.069	0.063	0.678	0.618	0.27
5.85	5.50	0.062	0.052	0.612	0.507	0.48
5.70	5.17	0.058	0.042	0.567	0.409	0.73
5.67	5.08	0.057	0.039	0.558	0.382	0.81

RUN	DM	Q	P	BA	Z
20	0.0222	0.0183	0.097	0.610	1.65

P/B	BA/B	FRAV	FRTOP
0.1212	0.762	0.0201	0.0148

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
9.83	9.79	0.184	0.183	1.896	1.884	0.05
9.32	9.28	0.168	0.167	1.736	1.723	0.05
8.83	8.78	0.153	0.152	1.582	1.566	0.07
8.32	8.27	0.138	0.136	1.421	1.406	0.07
7.84	7.76	0.123	0.121	1.270	1.245	0.11
7.33	7.23	0.108	0.105	1.110	1.079	0.14
7.10	6.97	0.101	0.097	1.038	0.997	0.18
6.86	6.68	0.093	0.088	0.962	0.906	0.25
6.75	6.41	0.090	0.080	0.928	0.821	0.47
6.43	6.13	0.080	0.071	0.827	0.733	0.41
6.26	5.78	0.075	0.060	0.774	0.623	0.66
6.15	5.49	0.072	0.052	0.739	0.531	0.91
6.14	5.39	0.071	0.048	0.736	0.500	1.03

RUN	DM	Q	P	BA	Z
21	0.0222	0.0261	0.097	0.610	1.65

P/B	BA/B	FRAV	FRTOP
0.1212	0.762	0.0287	0.0211

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.97	10.92	0.219	0.217	2.255	2.239	0.07
10.45	10.41	0.203	0.201	2.091	2.079	0.05
9.96	9.91	0.188	0.186	1.937	1.921	0.07
9.45	9.40	0.172	0.171	1.777	1.761	0.07
8.95	8.89	0.157	0.155	1.619	1.601	0.08
8.44	8.37	0.141	0.139	1.459	1.437	0.10
8.20	8.12	0.134	0.132	1.384	1.358	0.11
7.96	7.84	0.127	0.123	1.308	1.270	0.16
7.72	7.57	0.119	0.115	1.233	1.186	0.21
7.47	7.28	0.112	0.106	1.154	1.094	0.26
7.25	7.00	0.105	0.098	1.085	1.006	0.34
7.03	6.72	0.098	0.089	1.016	0.918	0.43
6.83	6.37	0.092	0.078	0.953	0.808	0.63
6.71	6.00	0.089	0.067	0.915	0.692	0.97
6.66	5.70	0.087	0.058	0.899	0.597	1.32

RUN	DM	Q	P	BA	Z
22	0.0222	0.0129	0.058	0.595	1.90

P/B	BA/B	FRAV	FRTOP
0.0724	0.744	0.0151	0.0121

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.15	10.12	0.194	0.193	3.342	3.326	0.04
9.64	9.62	0.178	0.177	3.074	3.063	0.03
8.90	8.85	0.155	0.154	2.684	2.658	0.07
8.39	8.36	0.140	0.139	2.416	2.400	0.04
7.90	7.85	0.125	0.123	2.158	2.132	0.07
7.40	7.37	0.110	0.109	1.895	1.879	0.04
6.90	6.83	0.094	0.092	1.632	1.595	0.10
6.39	6.30	0.079	0.076	1.363	1.316	0.12
6.17	6.02	0.072	0.068	1.247	1.168	0.21
5.96	5.75	0.066	0.059	1.137	1.026	0.29
5.80	5.46	0.061	0.051	1.053	0.874	0.47
5.68	5.18	0.057	0.042	0.989	0.726	0.69
5.66	5.10	0.057	0.040	0.979	0.684	0.77

RUN	DM	Q	P	BA	Z
23	0.0222	0.0179	0.058	0.595	1.90

P/B	BA/B	FRAV	FRTOP
0.0724	0.744	0.0209	0.0168

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
9.87	9.83	0.185	0.184	3.195	3.174	0.05
9.37	9.32	0.170	0.168	2.932	2.905	0.07
8.87	8.82	0.155	0.153	2.668	2.642	0.07
8.37	8.32	0.139	0.138	2.405	2.379	0.07
7.86	7.80	0.124	0.122	2.137	2.105	0.08
7.37	7.29	0.109	0.106	1.879	1.837	0.11
7.10	7.02	0.101	0.098	1.737	1.695	0.11
6.87	6.76	0.094	0.090	1.616	1.558	0.15
6.62	6.48	0.086	0.082	1.484	1.411	0.19
6.39	6.19	0.079	0.073	1.363	1.258	0.27
6.18	5.87	0.073	0.063	1.253	1.089	0.43
6.05	5.55	0.069	0.053	1.184	0.921	0.69
6.01	5.35	0.067	0.047	1.163	0.816	0.91
6.01	5.33	0.067	0.047	1.163	0.805	0.93

RUN	DM	Q	P	BA	Z
24	0.0222	0.0258	0.058	0.595	1.90

P/B	BA/B	FRAV	FRTOP
0.0724	0.744	0.0302	0.0242

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
10.42	10.37	0.202	0.200	3.484	3.458	0.07
9.91	9.87	0.186	0.185	3.216	3.195	0.05
9.41	9.36	0.171	0.169	2.953	2.926	0.07
8.90	8.84	0.155	0.154	2.684	2.653	0.08
8.39	8.33	0.140	0.138	2.416	2.384	0.08
7.89	7.79	0.125	0.122	2.153	2.100	0.14
7.63	7.53	0.117	0.114	2.016	1.963	0.14
7.37	7.25	0.109	0.105	1.879	1.816	0.16
7.13	6.96	0.101	0.096	1.753	1.663	0.23
6.89	6.77	0.094	0.091	1.626	1.563	0.16
6.67	6.36	0.087	0.078	1.511	1.347	0.43
6.52	6.00	0.083	0.067	1.432	1.158	0.71
6.46	5.79	0.081	0.061	1.400	1.047	0.92

RUN	DM	Q	P	BA	Z
25	0.0159	0.0584	0.089	1.430	3.38

P/B	BA/B	FRAV	FRTOP
0.0355	0.572	0.0076	0.0059

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.258	0.458	0.179	0.179	2.017	2.017	0.00
1.184	0.383	0.156	0.156	1.763	1.759	0.02
1.124	0.322	0.138	0.137	1.557	1.550	0.04
1.068	0.257	0.121	0.118	1.364	1.326	0.21
1.050	0.229	0.116	0.109	1.302	1.230	0.40
1.034	0.193	0.111	0.098	1.247	1.107	0.79
1.015	0.156	0.105	0.087	1.182	0.979	1.13
0.996	0.130	0.099	0.079	1.117	0.890	1.27
0.985	0.085	0.096	0.065	1.079	0.735	1.92

RUN	DM	Q	P	BA	Z
26	0.0159	0.0574	0.095	1.250	2.31

P/B	BA/B	FRAV	FRTOP
0.0382	0.500	0.0105	0.0085

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.284	0.484	0.187	0.187	1.958	1.958	0.00
1.196	0.391	0.160	0.158	1.677	1.661	0.10
1.162	0.357	0.150	0.148	1.569	1.553	0.10
1.117	0.305	0.136	0.132	1.425	1.387	0.23
1.060	0.242	0.119	0.113	1.243	1.185	0.35
1.037	0.190	0.112	0.097	1.169	1.019	0.90
1.020	0.148	0.106	0.084	1.115	0.885	1.38
1.012	0.128	0.104	0.078	1.089	0.821	1.61
0.993	0.066	0.098	0.059	1.029	0.623	2.43

RUN	DM	Q	P	BA	Z
27	0.0159	0.0405	0.095	1.250	2.31

P/B	BA/B	FRAV	FRTOP
0.0382	0.500	0.0074	0.0060

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.206	0.406	0.163	0.163	1.709	1.709	0.00
1.145	0.341	0.144	0.143	1.514	1.502	0.08
1.086	0.278	0.126	0.124	1.326	1.300	0.15
1.042	0.227	0.113	0.109	1.185	1.137	0.29
1.010	0.184	0.103	0.095	1.083	1.000	0.50
0.982	0.140	0.095	0.082	0.994	0.859	0.81
0.960	0.104	0.088	0.071	0.923	0.744	1.07
0.934	0.042	0.080	0.052	0.840	0.546	1.76

RUN	DM	Q	P	BA	Z
28	0.0159	0.0405	0.095	1.565	1.99

P/B	BA/B	FRAV	FRTOP
0.0382	0.626	0.0042	0.0036

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.168	0.366	0.151	0.151	1.588	1.581	0.04
1.128	0.327	0.139	0.139	1.460	1.457	0.02
1.096	0.292	0.130	0.128	1.358	1.345	0.08
1.049	0.241	0.115	0.113	1.208	1.182	0.15
1.020	0.204	0.106	0.101	1.115	1.064	0.31
0.969	0.140	0.091	0.082	0.952	0.859	0.56
0.951	0.114	0.085	0.074	0.895	0.776	0.71
0.910	0.042	0.073	0.052	0.764	0.546	1.30

RUN	DM	Q	P	BA	Z
29	0.0159	0.0287	0.095	1.565	1.99

P/B	BA/B	FRAV	FRTOP
0.0382	0.626	0.0030	0.0026

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.133	0.332	0.141	0.141	1.476	1.473	0.02
1.073	0.268	0.123	0.121	1.284	1.268	0.10
1.046	0.242	0.114	0.113	1.198	1.185	0.08
0.996	0.185	0.099	0.096	1.038	1.003	0.21
0.958	0.146	0.087	0.084	0.917	0.879	0.23
0.916	0.091	0.075	0.067	0.783	0.703	0.48
0.890	0.062	0.067	0.058	0.700	0.610	0.54
0.856	-0.006	0.056	0.037	0.591	0.393	1.19

RUN	DM	Q	P	BA	Z
30	0.0159	0.0578	0.095	1.565	1.99

P/B	BA/B	FRAV	FRTOP
0.0382	0.626	0.0060	0.0052

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.280	0.479	0.186	0.185	1.946	1.942	0.02
1.226	0.423	0.169	0.168	1.773	1.764	0.06
1.219	0.397	0.167	0.160	1.751	1.681	0.42
1.162	0.360	0.150	0.149	1.569	1.562	0.04
1.121	0.317	0.137	0.136	1.438	1.425	0.08
1.095	0.283	0.129	0.126	1.355	1.316	0.23
1.074	0.261	0.123	0.119	1.288	1.246	0.25
1.038	0.212	0.112	0.104	1.173	1.089	0.50
1.020	0.194	0.106	0.098	1.115	1.032	0.50
1.005	0.166	0.102	0.090	1.067	0.942	0.75
0.973	0.103	0.092	0.071	0.965	0.741	1.34



RUN	DM	Q	P	BA	Z
31	0.0159	0.0434	0.095	1.565	1.99

P/B	BA/B	FRAV	FRTOP
0.0382	0.626	0.0045	0.0039

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.182	0.382	0.156	0.156	1.633	1.633	0.00
1.144	0.344	0.144	0.144	1.511	1.511	0.00
1.118	0.316	0.136	0.136	1.428	1.422	0.04
1.070	0.253	0.122	0.116	1.275	1.220	0.33
1.038	0.229	0.112	0.109	1.173	1.144	0.17
1.020	0.203	0.106	0.101	1.115	1.061	0.33
0.989	0.161	0.097	0.088	1.016	0.927	0.54
0.978	0.139	0.094	0.082	0.981	0.856	0.75
0.952	0.098	0.086	0.069	0.898	0.725	1.04
0.920	0.055	0.076	0.056	0.796	0.588	1.25

RUN	DM	Q	P	BA	Z
32	0.0159	0.0278	0.100	1.875	1.89

P/B	BA/B	FRAV	FRTOP
0.0401	0.750	0.0018	0.0016

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.166	0.366	0.151	0.151	1.505	1.505	0.00
1.098	0.297	0.130	0.130	1.298	1.295	0.02
1.056	0.256	0.117	0.117	1.170	1.170	0.00
1.009	0.209	0.103	0.103	1.027	1.027	0.00
0.950	0.149	0.085	0.085	0.848	0.845	0.02
0.903	0.090	0.071	0.067	0.705	0.666	0.25
0.888	0.076	0.066	0.062	0.660	0.623	0.23
0.834	-0.006	0.050	0.037	0.495	0.374	0.77

RUN	DM	Q	P	BA	Z	
33	0.0159	0.0431	0.100	1.875	1.89	
	P/B	BA/B	FRAV	FRTOP		
	0.0401	0.750	0.0029	0.0025		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.253	0.453	0.177	0.177	1.769	1.769	0.00
1.197	0.396	0.160	0.160	1.599	1.596	0.02
1.150	0.347	0.146	0.145	1.456	1.447	0.06
1.111	0.309	0.134	0.134	1.337	1.331	0.04
1.064	0.256	0.120	0.117	1.195	1.170	0.15
1.025	0.208	0.108	0.103	1.076	1.024	0.33
0.985	0.165	0.096	0.090	0.954	0.894	0.38
0.955	0.134	0.087	0.080	0.863	0.799	0.40
0.899	0.054	0.069	0.056	0.693	0.556	0.86

RUN	DM	Q	P	BA	Z	
34	0.0159	0.0581	0.100	1.875	1.89	
	P/B	BA/B	FRAV	FRTOP		
	0.0401	0.750	0.0039	0.0034		
PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.330	0.527	0.201	0.200	2.003	1.994	0.06
1.237	0.437	0.173	0.173	1.720	1.720	0.00
1.173	0.368	0.153	0.151	1.526	1.511	0.10
1.113	0.306	0.135	0.133	1.343	1.322	0.13
1.067	0.257	0.121	0.118	1.204	1.173	0.19
1.037	0.218	0.112	0.106	1.112	1.055	0.36
1.004	0.172	0.101	0.092	1.012	0.915	0.61
0.954	0.109	0.086	0.073	0.860	0.723	0.86

RUN	DM	Q	P	BA	Z
35	0.0159	0.0285	0.100	1.250	1.99

P/B	BA/B	FRAV	FRTOP
0.0401	0.500	0.0052	0.0043

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.177	0.376	0.154	0.154	1.538	1.535	0.02
1.101	0.297	0.131	0.130	1.307	1.295	0.08
1.061	0.253	0.119	0.116	1.185	1.161	0.15
1.025	0.210	0.108	0.103	1.076	1.030	0.29
0.977	0.154	0.093	0.086	0.930	0.860	0.44
0.954	0.133	0.086	0.080	0.860	0.796	0.40
0.926	0.097	0.078	0.069	0.775	0.687	0.56
0.914	0.061	0.074	0.058	0.739	0.578	1.02
0.886	0.014	0.066	0.044	0.653	0.435	1.38
0.879	-0.028	0.063	0.031	0.632	0.307	2.05

RUN	DM	Q	P	BA	Z
36	0.0159	0.0430	0.100	1.250	1.99

P/B	BA/B	FRAV	FRTOP
0.0401	0.500	0.0079	0.0065

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.261	0.461	0.180	0.180	1.793	1.793	0.00
1.191	0.389	0.158	0.158	1.581	1.574	0.04
1.141	0.335	0.143	0.141	1.429	1.410	0.12
1.086	0.270	0.126	0.122	1.261	1.213	0.31
1.051	0.229	0.116	0.109	1.155	1.088	0.42
1.026	0.191	0.108	0.098	1.079	0.973	0.67
0.991	0.141	0.098	0.082	0.973	0.821	0.96
0.973	0.117	0.092	0.075	0.918	0.748	1.07
0.954	0.084	0.086	0.065	0.860	0.647	1.34
0.936	0.005	0.081	0.041	0.805	0.407	2.51

RUN	DM	Q	P	BA	Z
37	0.0159	0.0575	0.100	1.250	1.99

P/B	BA/B	FRAV	FRTOP
0.0401	0.500	0.0105	0.0087

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.314	0.512	0.196	0.195	1.954	1.948	0.04
1.256	0.456	0.178	0.178	1.778	1.778	0.00
1.217	0.412	0.166	0.165	1.660	1.644	0.10
1.184	0.377	0.156	0.154	1.559	1.538	0.13
1.153	0.342	0.147	0.144	1.465	1.432	0.21
1.120	0.308	0.137	0.133	1.365	1.328	0.23
1.090	0.267	0.128	0.121	1.274	1.204	0.44
1.070	0.231	0.122	0.110	1.213	1.094	0.75
1.055	0.211	0.117	0.104	1.167	1.033	0.84
1.041	0.182	0.113	0.095	1.125	0.945	1.13
1.023	0.146	0.107	0.084	1.070	0.836	1.48
1.011	0.080	0.104	0.064	1.033	0.635	2.51

RUN	DM	Q	P	BA	Z
38	0.0071	0.0406	0.100	1.250	1.99

P/B	BA/B	FRAV	FRTOP
0.0401	0.500	0.0074	0.0061

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.281	0.482	0.186	0.186	1.854	1.857	-0.04
1.249	0.449	0.176	0.176	1.757	1.757	0.00
1.222	0.421	0.168	0.168	1.675	1.672	0.04
1.143	0.337	0.144	0.142	1.435	1.416	0.26
1.120	0.315	0.137	0.135	1.365	1.350	0.21
1.099	0.287	0.130	0.127	1.301	1.264	0.51
1.080	0.263	0.125	0.119	1.243	1.191	0.73
1.055	0.223	0.117	0.107	1.167	1.070	1.37 *
1.024	0.177	0.108	0.093	1.073	0.930	2.01
0.994	0.140	0.098	0.082	0.982	0.818	2.31

RUN	DM	Q	P	BA	Z
40	0.0071	0.0570	0.100	1.250	1.99

P/B	BA/B	FRAV	FRTOP
0.0401	0.500	0.0104	0.0086

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.352	0.550	0.208	0.207	2.070	2.064	0.09
1.302	0.498	0.192	0.191	1.918	1.906	0.17
1.278	0.474	0.185	0.184	1.845	1.833	0.17
1.252	0.448	0.177	0.176	1.766	1.754	0.17
1.236	0.426	0.172	0.169	1.717	1.687	0.43
1.204	0.394	0.162	0.159	1.620	1.590	0.43
1.178	0.370	0.155	0.152	1.541	1.517	0.34
1.161	0.352	0.149	0.147	1.489	1.462	0.38
1.139	0.327	0.143	0.139	1.422	1.386	0.51
1.123	0.301	0.138	0.131	1.374	1.307	0.94
1.106	0.280	0.133	0.125	1.322	1.243	1.11
1.086	0.250	0.126	0.116	1.261	1.152	1.54 *

RUN	DM	Q	P	BA	Z
41	0.0071	0.0405	0.082	1.250	1.83

P/B	BA/B	FRAV	FRTOP
0.0328	0.500	0.0074	0.0064

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.317	0.515	0.197	0.196	2.401	2.394	0.09
1.252	0.452	0.177	0.177	2.160	2.160	0.00
1.224	0.422	0.169	0.168	2.056	2.048	0.09
1.185	0.380	0.157	0.155	1.911	1.892	0.21
1.154	0.349	0.147	0.146	1.796	1.777	0.21
1.127	0.324	0.139	0.138	1.695	1.684	0.13
1.119	0.314	0.137	0.135	1.665	1.647	0.21
1.084	0.276	0.126	0.123	1.535	1.506	0.34
1.058	0.247	0.118	0.115	1.439	1.398	0.47
1.036	0.224	0.111	0.108	1.357	1.312	0.51
1.026	0.206	0.108	0.102	1.320	1.245	0.85
1.008	0.178	0.103	0.094	1.253	1.141	1.28 *
0.982	0.128	0.095	0.078	1.156	0.955	2.31

RUN	DM	Q	P	BA	Z
42	0.0071	0.0429	0.075	1.250	2.00

P/B	BA/B	FRAV	FRTOP
0.0300	0.500	0.0078	0.0068

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.225	0.425	0.169	0.169	2.252	2.252	0.00
1.182	0.381	0.156	0.155	2.077	2.073	0.04
1.148	0.347	0.145	0.145	1.939	1.935	0.04
1.103	0.303	0.132	0.132	1.756	1.756	0.00
1.084	0.281	0.126	0.125	1.679	1.667	0.13
1.051	0.247	0.116	0.115	1.545	1.528	0.17
1.037	0.228	0.112	0.109	1.488	1.451	0.38
1.019	0.209	0.106	0.103	1.415	1.374	0.43
1.004	0.187	0.101	0.096	1.354	1.285	0.73
0.989	0.154	0.097	0.086	1.293	1.150	1.49 *

RUN	DM	Q	P	BA	Z
43	0.0071	0.0572	0.075	1.250	2.00

P/B	BA/B	FRAV	FRTOP
0.0300	0.500	0.0105	0.0090

PG1	PG3	Y1	Y2	Y1/P	Y2/P	(Y1-Y2)/dm
1.342	0.542	0.205	0.205	2.728	2.728	0.00
1.302	0.502	0.192	0.192	2.565	2.565	0.00
1.272	0.471	0.183	0.183	2.443	2.439	0.04
1.240	0.440	0.173	0.173	2.313	2.313	0.00
1.204	0.399	0.162	0.161	2.167	2.146	0.21
1.184	0.383	0.156	0.156	2.085	2.081	0.04
1.174	0.373	0.153	0.153	2.045	2.041	0.04
1.146	0.343	0.145	0.144	1.931	1.919	0.13
1.121	0.316	0.137	0.136	1.829	1.809	0.21
1.102	0.293	0.131	0.129	1.752	1.715	0.38
1.080	0.275	0.125	0.123	1.663	1.642	0.21
1.060	0.248	0.119	0.115	1.581	1.533	0.51
1.041	0.220	0.113	0.106	1.504	1.419	0.90
1.030	0.204	0.109	0.101	1.459	1.354	1.11 *
1.022	0.188	0.107	0.097	1.427	1.289	1.45
1.006	0.147	0.102	0.084	1.362	1.122	2.52
0.978	0.061	0.094	0.058	1.248	0.772	4.99